Computation over Groups
Some Special Features

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Computation over Groups
Some Special Features

Our goal:

- Investigate possible relationships between the classes

$$\mathrm{P}_G \subseteq \mathrm{DNP}_G \subseteq \mathrm{NP}_G,$$

$$\mathrm{P}_G{}^A \subseteq \mathrm{DNP}_G{}^A \subseteq \mathrm{NP}_G{}^A.$$

- Investigate special features resulting from the existence of only one constant.

Computation over Groups
Some Special Features

1. The uniform model of computation
2. A finite group with $\mathrm{P} \neq \mathrm{NP}$
3. Groups and oracles with $\mathrm{P}^A = \mathrm{NP}^A$
4. Groups and oracles with $\mathrm{P}^A \neq \mathrm{DNP}^A$ or $\mathrm{DNP}^A \neq \mathrm{NP}^A$
5. Embedding of a group into a structure with $\mathrm{P} = \mathrm{NP}$

The uniform model of computation

A structure: $\Sigma = (U\,;\, c_1, \ldots, c_u\,;\, f_1, \ldots, f_v\,;\, R_1, \ldots, R_w, =)$

Computation: $l\colon\ Z_k := f_j(Z_{k_1}, \ldots, Z_{k_{m_j}});$
$l\colon\ Z_k := c_j;$

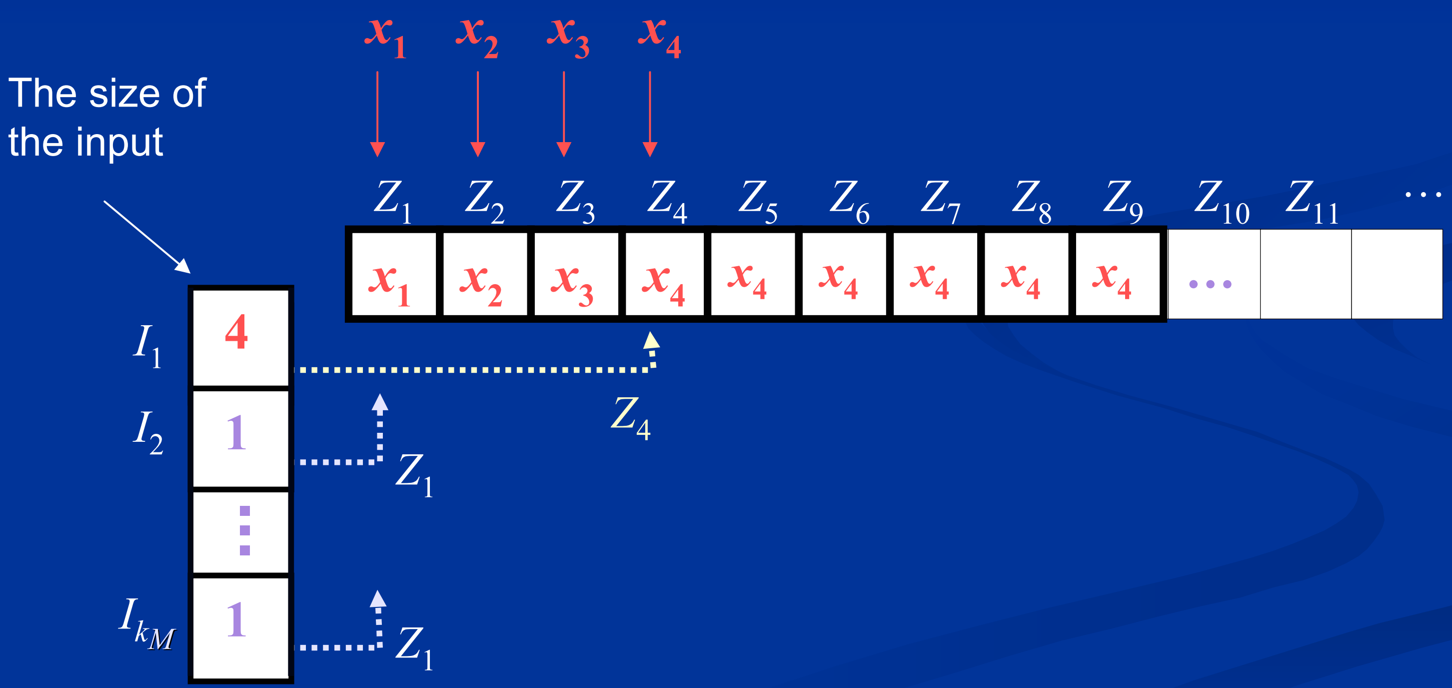
Branching: $l\colon$ if $R_j(Z_{k_1}, \ldots, Z_{k_{n_j}})$ then goto l_1 else goto l_2;
$l\colon$ if $Z_k = Z_j$ then goto l_1 else goto l_2;

Copy: $l\colon\ Z_{I_k} := Z_{I_j};$

Index computation: $I_k := 1;\ \ I_k := I_k + 1;$ if $I_k = I_j$ then goto l_1 else goto l_2;

The machine and the input

The input: $(Z_1, \ldots, Z_n) := (x_1, \ldots, x_n);\quad I_1 := n;\ I_2 := 1; \ldots\ I_{k_M} := 1;$



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Computation in polynomial time

Computation in polynomial time:

For any machine M there is some polynomial p_M such that

$$M \text{ halts for } \boldsymbol{x} = (x_1, \ldots, x_n) \text{ within } p_M(n) \text{ steps.}$$

The execution of one operation is one time unit.

⇨ $P_\Sigma \subseteq DEC_\Sigma$ (P_Σ ≙ problems are decidable in polynomial time)

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The non-deterministic instructions

The non-determinism of the first kind:

goto l_1 else goto l_2 ;

⇨ $P_\Sigma \subseteq DNP_\Sigma \subseteq DEC_\Sigma$

The non-determinism of the second kind:

guess(Z_k) ; Arbitrary elements can be guessed!

⇨ $P_\Sigma \subseteq NP_\Sigma$ If Σ contains two elements, then $DNP_\Sigma \subseteq NP_\Sigma$.

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Some $P_\Sigma \overset{?}{=} NP_\Sigma$ problems for several structures

The uniform model of computation
A finite group with $P \neq NP$
Groups and oracles with $P^A = NP^A$
Groups and oracles with $P^A \neq DNP^A$ or $DNP^A \neq NP^A$
Embedding of a group into a structure with $P = NP$

Σ	$P_\Sigma = DNP_\Sigma$?	$DNP_\Sigma = NP_\Sigma$?
$(\mathbb{C}; \mathbb{C}; +, -, \cdot; =)$	**?**	**?**
$(\mathbb{R}; \mathbb{R}; +, -, \cdot; \leq)$	**?**	**?**
$(\mathbb{R}; \mathbb{R}; +, -, \cdot; =)$	**?**	no ($\leq$)
$(\mathbb{R}; \mathbb{R}; +, -; \leq)$	**?**	yes (Koiran)
$(\mathbb{R}; \mathbb{R}; +, -; =)$	no (Meer / Koiran)	yes (Koiran)
$(\mathbb{Z}; \mathbb{Z}; +, -; \leq)$	**?**	no (even integers)
$(\mathbb{Z}; \mathbb{Z}; +, -; =)$	no (inf. abl. groups)	no (even integers)
infinite ablian groups	no	no / yes

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Instructions for groups

Let $(G; e; \circ; =)$ be a group.

Computation:

(1) $l\colon\ Z_k := Z_i \circ Z_j\,;$

(2) without parameters: $l\colon\ Z_k := e\,;$ ⇨ $(G; e; \circ; =)$-machines

with parameters $g \in G$: $l\colon\ Z_k := g\,;$ ⇨ $(G; G; \circ; =)$-machines

Branching: $l\colon$ if $Z_k = Z_j$ then goto l_1 else goto l_2 ;

Copy: $l\colon\ Z_{I_k} := Z_{I_j}\,;$

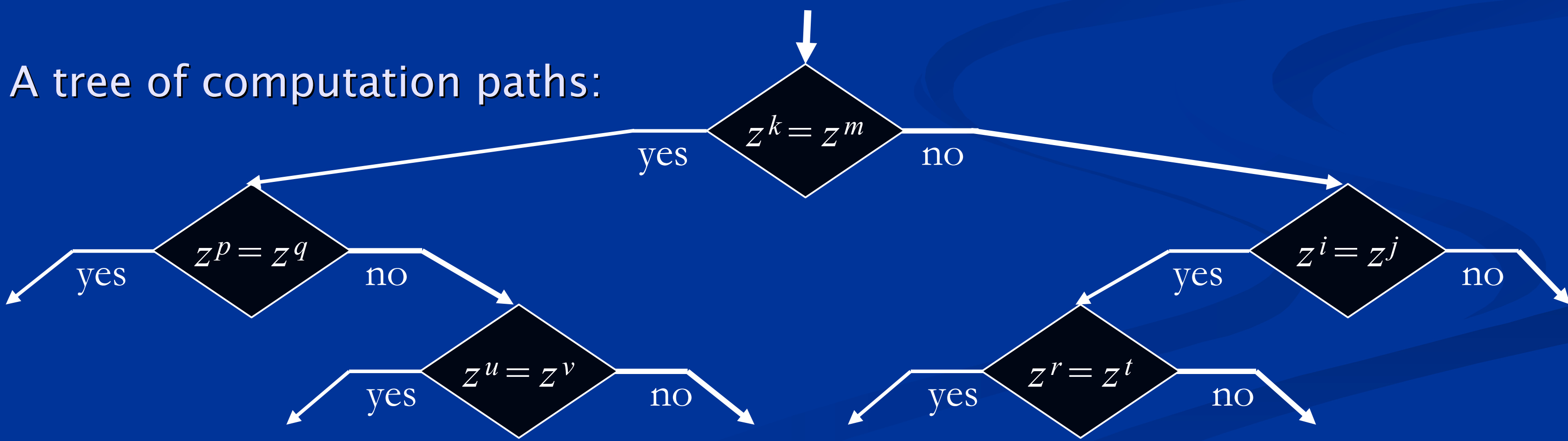
Decidability by $(G; e; \circ; =)$-machines

We cannot separate
elements of the same order without additional parameters.

$A \subseteq G$,

$$x \in A, \quad y \notin A, \quad \text{order}(x) = \text{order}(y) \quad \Rightarrow \quad A \notin \text{DEC}_G.$$

A tree of computation paths:



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The dihedral group

$D(2,4) =$

$(\{e, r, r^2, r^3, s, rs, r^2s, r^3s\}, \circ)$

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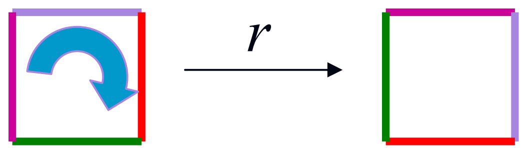
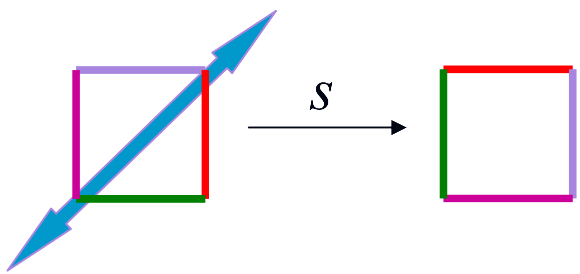
$\circ$	e	r	r^2	r^3	s	rs	r^2	r^3s
e	e	r	r^2	r^3	s	rs	r^2s	r^3s
r	r	r^2	r^3	e	rs	r^2s	r^3s	s
r^2	r^2	r^3	e	r	r^2s	r^3s	s	rs
r^3	r^3	e	r	r^2	r^3s	s	rs	r^2s
s	s	r^3s	r^2s	rs	e	r^3	r^2	r
rs	rs	s	r^3s	r^2s	r	e	r^3	r^2
r^2s	r^2s	rs	s	r^3s	r^2	r	e	r^3
r^3s	r^3s	r^2s	rs	s	r^3	r^2	r	e

Rotation of a square:



Reflection of a square:



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$\mathbf{P}_{D\,(2,4)} \neq \mathbf{NP}_{D\,(2,4)}$

$\circ$	e	r	r^2	r^3	s	rs	r^2	r^3s
e	e	r	r^2	r^3	s	rs	r^2s	r^3s
r	r	r^2	r^3	e	rs	r^2s	r^3s	s
r^2	r^2	r^3	e	r	r^2s	r^3s	s	rs
r^3	r^3	e	r	r^2	r^3s	s	rs	r^2s
s	s	r^3s	r^2s	rs	e	r^3	r^2	r
rs	rs	s	r^3s	r^2s	r	e	r^3	r^2
r^2s	r^2s	rs	s	r^3s	r^2	r	e	r^3
r^3s	r^3s	r^2s	rs	s	r^3	r^2	r	e

$$A = \{x \mid \exists y\,(y \circ x \neq x \circ y)\} = \{r, r^3, s, rs, r^2s, r^3s\}$$

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$\mathbf{P}_{D\,(2,4)} \neq \mathbf{NP}_{D\,(2,4)}$

The uniform model of computation
A finite group with P ≠ NP
Groups and oracles with $P^A = NP^A$
Groups and oracles with $P^A \neq DNP^A$ or $DNP^A \neq NP^A$
Embedding of a group into a structure with P = NP

$\circ$	e	r	r^2	r^3	s	rs	r^2	r^3s
e	e	r	r^2	r^3	s	rs	r^2s	r^3s
r	r	r^2	r^3	e	rs	r^2s	r^3s	s
r^2	r^2	r^3	e	r	r^2s	r^3s	s	rs
r^3	r^3	e	r	r^2	r^3s	s	rs	r^2s
s	s	r^3s	r^2s	rs	e	r^3	r^2	r
rs	rs	s	r^3s	r^2s	r	e	r^3	r^2
r^2s	r^2s	rs	s	r^3s	r^2	r	e	r^3
r^3s	r^3s	r^2s	rs	s	r^3	r^2	r	e

$$A = \{x \mid \exists y\,(y \circ x \neq x \circ y)\} = \{r, r^3, s, rs, r^2s, r^3s\}$$

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The uniform model of computation
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$\mathrm{P}_{D(2,4)} \neq \mathrm{NP}_{D(2,4)}$

$\circ$	e	r	r^2	r^3	s	rs	r^2	r^3s
e	e	r	r^2	r^3	s	rs	r^2s	r^3s
r	r	r^2	r^3	e	rs	r^2s	r^3s	s
r^2	r^2	r^3	e	r	r^2s	r^3s	s	rs
r^3	r^3	e	r	r^2	r^3s	s	rs	r^2s
s	s	r^3s	r^2s	rs	e	r^3	r^2	r
rs	rs	s	r^3s	r^2s	r	e	r^3	r^2
r^2s	r^2s	rs	s	r^3s	r^2	r	e	r^3
r^3s	r^3s	r^2s	rs	s	r^3	r^2	r	e

1. $A \in \mathrm{NP}_{D(2,4)}$
2. $A \notin \mathrm{P}_{D(2,4)}$

since

$s \in A$

$r^2 \notin A$

and

$\mathrm{order}(s) = \mathrm{order}(r^2) = 2$

$$A = \{x \mid \exists y\,(y \circ x \neq x \circ y)\} = \{r, r^3, s, rs, r^2s, r^3s\}$$

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$P_{D(2,4)} \neq NP_{D(2,4)}$

$\circ$	e	r	r^2	r^3	s	rs	r^2	r^3s
e	e	r	r^2	r^3	s	rs	r^2s	r^3s
r	r	r^2	r^3	e	rs	r^2s	r^3s	s
r^2	r^2	r^3	e	r	r^2s	r^3s	s	rs
r^3	r^3	e	r	r^2	r^3s	s	rs	r^2s
s	s	r^3s	r^2s	rs	e	r^3	r^2	r
rs	rs	s	r^3s	r^2s	r	e	r^3	r^2
r^2s	r^2s	rs	s	r^3s	r^2	r	e	r^3
r^3s	r^3s	r^2s	rs	s	r^3	r^2	r	e

1. $A \in NP_{D(2,4)}$
2. $A \notin P_{D(2,4)}$

since

$s \in A$

$r^2 \notin A$

and

$\mathrm{order}(s) = \mathrm{order}(r^2) = 2$

$$A = \{x \mid \exists y\, (y \circ x \neq x \circ y)\} = \{r, r^3, s, rs, r^2s, r^3s\}$$

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Oracle machines

Oracle query:

$$l\text{: if } (\underbrace{Z_1, \ldots, Z_{I_1}}) \in A \quad \text{then goto } l_1 \text{ else goto } l_2\,;$$

The length can be computed by $I_1 := 1; \quad I_1 := I_1 + 1; \ldots.$

$$A \subseteq G^\infty = \cup_{n \geq 1} G^n$$

We will define oracles such that

$DNP_G^{\,O} = NP_G^{\,O}$ and $DNP_G \neq NP_G$ (e.g. for $G = (\mathbb{Z}\,;\,0\,;\,+\,;\,=)$)

$DNP_{\bar{G}}^{\,Q} \neq NP_{\bar{G}}^{\,Q}$ and $DNP_{\bar{G}} = NP_{\bar{G}}$ (e.g. for $\bar{G} = (\mathbb{R}\,;\,\mathbb{R}\,;\,+\,;\,=)$)

(cp. also Baker, Gill, and Solovay; Emerson; ... for Turing machines...)

An oracle O with $P_{\bar{G}}^{O} = NP_{\bar{G}}^{O}$

A universal oracle for $\bar{G} = (G; G; \circ; =)$ with $a, b \in G$:

$$O = O_{\bar{G}} = \{(\boldsymbol{x}, Code(M), \underbrace{e, \ldots, e}_{t}) \in G^{\infty} \mid Code(M) \in \{a, b\}^{\infty}$$

$$\& \quad M \text{ is an } NP_{\bar{G}}^{O}\text{-machine}$$

$$\& \quad \underbrace{M(\boldsymbol{x}) \downarrow^{t}}\ \}$$

M accepts input $\boldsymbol{x} = (x_1, \ldots, x_n) \in G^{\infty}$ within t steps

Proposition: $P_{\bar{G}}^{O} = DNP_{\bar{G}}^{O} = NP_{\bar{G}}^{O}$ (with parameters).

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An oracle O with $P_G^{\,O} = NP_G^{\,O}$?

$O = O_G = \{(\boldsymbol{x}, Code(M), e, \ldots, e) \in G^\infty \mid Code(M) \in \{a, b\}^\infty$

& M is an $NP_G^{\,O}$-machine

& $M(\boldsymbol{x}) \downarrow^t \}$

$A \subseteq \{e\}^\infty$ cannot be reduced to O
if $Code(M_A) \in \{a, b\}^\infty$ cannot be computed from the input.

⇒ O is not $NP_G^{\,O}$-complete (without parameters).

An oracle O with $P_G^O = NP_G^O$

A universal oracle for G:

$$O = O_G = \cup_{a,b \in G} \{(x, Code(M), a, \ldots, a) \in G^\infty \mid Code(M) \in \{a, b\}^\infty$$

$$\& \quad M \text{ is an } NP_G^O\text{-machine}$$

$$\& \quad M(x) \downarrow^t \}$$

$$\cup \quad \{e \in \{e\}^\infty \mid e \text{ is the code of a } ((\underbrace{e, \ldots, e}_{x}), M, t)$$

$$\& \quad M(x) \downarrow^t \}$$

$\Rightarrow \quad A \subseteq \{e\}^\infty$ can be reduced only to $O_G \cap \{e\}^\infty$.

Proposition: $P_G^O = DNP_G^O = NP_G^O$ (without parameters).

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An oracle Q with $DNP_G^{\,Q} \neq NP_G^{\,Q}$

For any oracle $B \subseteq G^\infty$,

let N_i^B be the $DNP_G{}^B$-machine

- executing $p_i(n)$ instructions of program P_i for any $\boldsymbol{x} \in G^n$.

$V_0 = \emptyset$, $m_0 = 0$.

Stage $i \geq 1$: Let $n_i > m_{i-1}$, $m_i = 2^{n_i}$, $p_i(n_i) + n_i < m_i$.

$W_i = \cup_{j<i} V_j$

$V_i = \{\boldsymbol{x} \in G^{n_i} \mid N_i^{W_i}$ does not accept $(e,\ldots,e) \in G^{n_i}$

$\&$ $\boldsymbol{x}$ is not queried by $N_i^{W_i}$ on $(e,\ldots,e) \in G^{n_i}\}$

We use diagonalization techniques.

$Q = Q_G = \cup_{i \geq 1} W_i$ $\qquad$ $L = \{\boldsymbol{y} \mid (\exists\, i \geq 1)(\boldsymbol{y} \in G^{n_i} \;\&\; V_i \neq \emptyset)\}$

Proposition: If G is infinite, then $DNP_G{}^Q \neq NP_G{}^Q$ (without parameters).

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An oracle Q with $\mathrm{DNP}_{\bar{G}}^{Q} \neq \mathrm{NP}_{\bar{G}}^{Q}$

If G is countable, for any oracle $B \subseteq G^\infty$,

let N_i^B be the $\mathrm{DNP}_{\bar{G}}^{B}$-machine

- executing $p_i(n)$ instructions of program P_i for any $x \in G^n$.

$V_0 = \emptyset$, $m_0 = 0$.

Stage $i \geq 1$: Let $n_i > m_{i-1}$, $m_i = 2^{n_i}$, $p_i(n_i) + n_i < m_i$.

$W_i = \cup_{j<i} V_j$

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& x is not queried by $N_i^{W_i}$ on $(e,\ldots,e) \in G^{n_i}\}$

$Q = Q_{\bar{G}} = \cup_{i \geq 1} W_i$ $\qquad$ $L = \{y \mid (\exists\, i \geq 1)(y \in G^{n_i}$ & $V_i \neq \emptyset)\}$

Proposition: If G is infinite and countable, then $\mathrm{DNP}_{\bar{G}}^{Q} \neq \mathrm{NP}_{\bar{G}}^{Q}$ (with p.).

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An oracle Q with $P_{\bar{G}}^{Q} \neq DNP_{\bar{G}}^{Q}$

If G is countable and $|G| \geq 2$, for any oracle $B \subseteq G^\infty$,

let N_i^B be the $P_{\bar{G}}^{B}$-machine

- executing $p_i(n)$ instructions of program P_i for any $x \in G^n$.

$V_0 = \emptyset$, $m_0 = 0$.

Stage $i \geq 1$: Let $n_i > m_{i-1}$, $m_i = 2^{n_i}$, $p_i(n_i) + n_i < m_i$.

$W_i = \cup_{j<i} V_j$

$V_i = \{x \in G^{n_i} \mid N_i^{W_i}$ does not accept $(e,\ldots,e) \in G^{n_i}$

$\& \;\; x$ is not queried by $N_i^{W_i}$ on $(e,\ldots,e) \in G^{n_i}\}$

$Q = Q_{\bar{G}} = \cup_{i \geq 1} W_i$ $\qquad$ $L = \{y \mid (\exists\, i \geq 1)(y \in G^{n_i} \;\&\; V_i \neq \emptyset)\}$

Proposition: If G is countable and $|G| \geq 2$, then $P_{\bar{G}}^{Q} \neq DNP_{\bar{G}}^{Q}$ (with p.).

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An oracle Q with $P_G^Q \neq DNP_G^Q$

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If $a, b \in G$, for any oracle $B \subseteq G^\infty$,

let N_i^B be the P_G^B-machine

- executing $p_i(n)$ instructions of program P_i for any $x \in G^n$.

$V_0 = \emptyset$, $m_0 = 0$.
Stage $i \geq 1$: Let $n_i > m_{i-1}$, $m_i = 2^{n_i}$, $p_i(n_i) + n_i + 2 < m_i$.
$W_i = \cup_{j<i} V_j$

$V_i = \{x \in \{a, b\}^{n_i} \setminus \{a\}^{n_i} \mid N_i^{W_i}$ does not accept $(a, b,\dots, b) \in G^{n_i}$

& x is not queried by $N_i^{W_i}$ on $(a, b,\dots, b) \in G^{n_i}\}$

$Q = Q_G = \cup_{i\geq 1} W_i$ $\qquad$ $L = \{y \mid (\exists\, i \geq 1)(y \in \{a, b\}^{n_i} \setminus \{a\}^{n_i}\ \&\ V_i \neq \emptyset)\}$

Proposition: If $a, b \in G$, then $P_G^Q \neq DNP_G^Q$ (without parameters).

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An oracle Q with $\mathrm{DNP}_{\bar{G}}^{Q} \neq \mathrm{NP}_{\bar{G}}^{Q}$

If G is not countable, for suitable codes $\boldsymbol{u} \in U \subseteq G^\infty$ and any oracle $B \subseteq G^\infty$,

let $N_{\boldsymbol{u}}^B$ be the $\mathrm{DNP}_{\bar{G}}^B$-machine

- executing $p_{\boldsymbol{u}}(n)$ instructions of program $P_{\boldsymbol{u}}$ for any $\boldsymbol{x} \in G^n$.

$V_0 = \emptyset$.

Stage $i \geq 1$:

$K_i = \{\boldsymbol{u} \in U \mid (\forall j \geq i)(\forall B \subseteq G^\infty)$
$(N_{\boldsymbol{u}}^B$ does not compute or use the value a^j on $\boldsymbol{u})\}$

$W_i = \cup_{k<i} V_k$

$V_i = \{(a^{i+1}, \boldsymbol{u}) \mid \boldsymbol{u} \in K_i \;\&\; N_{\boldsymbol{u}}^{W_i}$ does not accept $\boldsymbol{u}\}$

$Q = Q_{\bar{G}} = \cup_{i \geq 1} W_i$ $\qquad L = \{ \boldsymbol{y} \mid (\exists\, n \geq 2)((a^n, \boldsymbol{y}) \in Q_{\bar{G}})\}$

Proposition:
If G contains a with $\mathrm{order}(a) = \infty$, then $\mathrm{DNP}_{\bar{G}}^{Q} \neq \mathrm{NP}_{\bar{G}}^{Q}$ (with parameters).

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An oracle Q with $P_{\bar{G}}^{Q} \neq DNP_{\bar{G}}^{Q}$?

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G is not countable and G is not abelian.

?

(If G is infinite and abelian, then $P_{\bar{G}}^{\varnothing} \neq DNP_{\bar{G}}^{\varnothing}$.)

Is there an oracle with $P_{\bar{G}}^{Q} \neq DNP_{\bar{G}}^{Q}$ (if parameters are allowed) ?

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Embedding of a group into a structure with P = NP

Theorem: $(G; e; \circ; =)$ can be embedded into

$$\bar{G}_R^* = (G^*;\ A \cup \{\varepsilon\};\ \circ, \mathrm{add}, \mathrm{sub}_\mathrm{l}, \mathrm{sub}_\mathrm{r};\ =, R)$$

such that $P_{\bar{G}_R^*} = NP_{\bar{G}_R^*}$, if $\{g_1, g_2\} \subseteq A \subseteq G$. (CiE 2006 / 2007)

Disadvantage:

- The axioms of groups are satisfied only on G.
- $A = \{e\}$ is not sufficient.

⇨ A more natural extension Σ: derived from binary (searching) trees

- with $P_\Sigma = DNP_\Sigma$ where the test of identity is possible,
- with $P_\Sigma = NP_\Sigma$ where the identity is decidable.

cp. also C. Gaßner: Über die Konstruktion von Strukturen endlicher Signatur mit P = NP. Preprint 1/2004, Preprint-Reihe Mathematik, Greifswald.

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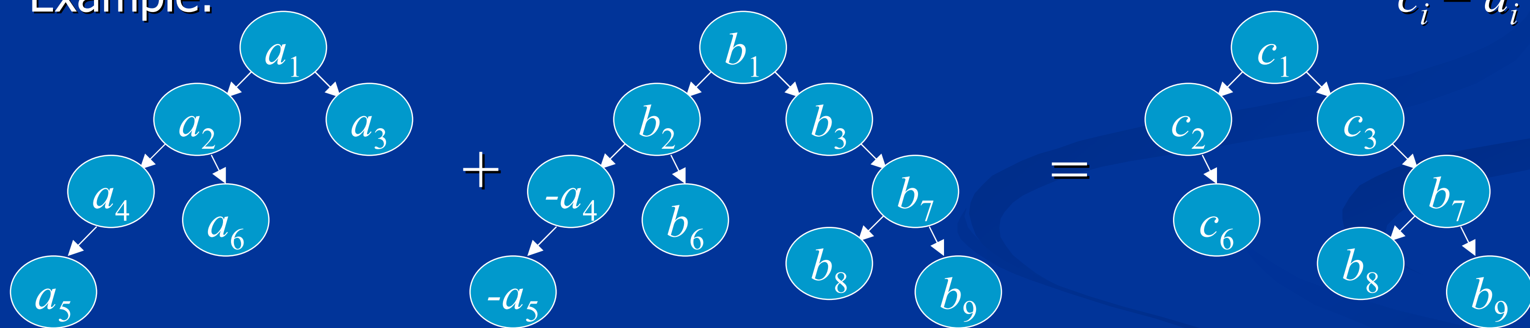
Embedding of a group into a structure with P = DNP

Theorem: $(G; e; \circ; =)$ can be embedded into

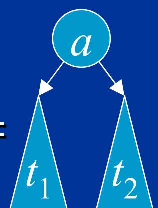
$$G_R^{\mathrm{tree}} = (\mathrm{tree}(G);\ \mathrm{nil};\ \circ, \mathrm{concat}, \mathrm{root}, \mathrm{sub}_{\mathrm{l}}, \mathrm{sub}_{\mathrm{r}};\ =, R)$$

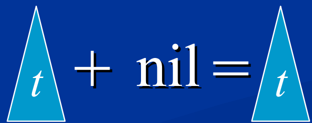
satisfying the axioms of groups and $\mathrm{P}_{G_R^{\mathrm{tree}}} = \mathrm{DNP}_{G_R^{\mathrm{tree}}}$.

Example: $c_i = a_i + b_i$



$\mathrm{concat}(a, t_1, t_2) =$





R is satisfied by the codes of the elements of a universal DNP-oracle.

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Summary

We know

- groups G (e.g. $(\mathbb{Z}\,;0\,;+\,;=)$ and $(\mathbb{R}\,;\mathbb{R}\,;+\,;\,=)$, respectively) with

$$\mathrm{P}_G^{\,O} = \mathrm{DNP}_G^{\,O} = \mathrm{NP}_G^{\,O} \quad \& \quad \mathrm{P}_G \neq \mathrm{DNP}_G \neq \mathrm{NP}_G$$

$$\mathrm{P}_G^{\,O} = \mathrm{DNP}_G^{\,O} \quad \& \quad \mathrm{DNP}_G^{\,Q} \neq \mathrm{NP}_G^{\,Q} \quad \& \quad \mathrm{P}_G \neq \mathrm{DNP}_G = \mathrm{NP}_G$$

- structures Σ (e.g. trees over a group with identity) satisfying the axioms of groups with

$$\mathrm{P}_\Sigma^{\,O} \neq \mathrm{DNP}_\Sigma^{\,O} \quad \& \quad \mathrm{DNP}_\Sigma^{\,Q} = \mathrm{NP}_\Sigma^{\,Q} \quad \& \quad \mathrm{P}_\Sigma = \mathrm{DNP}_\Sigma \neq \mathrm{NP}_\Sigma$$

- structures Σ (e.g. $(\mathbb{Z}^*;\,\mathbb{Z}\cup\{\varepsilon\};\,+,\,\mathrm{add}\,,\,\mathrm{sub}_l\,,\,\mathrm{sub}_r\,;=\,,\,R)$) with

$$\mathrm{P}_\Sigma^{\,O} \neq \mathrm{DNP}_\Sigma^{\,O} \quad \& \quad \mathrm{DNP}_\Sigma^{\,Q} \neq \mathrm{NP}_\Sigma^{\,Q} \quad \& \quad \mathrm{P}_\Sigma = \mathrm{DNP}_\Sigma = \mathrm{NP}_\Sigma$$

⇨ oracles are not very helpful for solving $\mathrm{P} \stackrel{?}{=} \mathrm{NP}$ problems.

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Computation over Groups

Some Special Features

Thank you for your attention!

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