▸ RUPERT HÖLZL, CHRISTOPHER C. PORTER, *Randomness for computable measures and initial segment complexity.*
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According to the Levin-Schnorr theorem, a sequence $X \in 2^\omega$ is Martin-Löf random with respect to the Lebesgue measure if and only if $\mathrm{K}(X \upharpoonright n) \geq n - O(1)$ for every n, where K denotes prefix-free Kolmogorov complexity. Roughly, this means that the Martin-Löf random sequences are precisely those sequences with high initial segment complexity. It is well-known that the Levin-Schnorr theorem can be extended to proper sequences, that is, sequences that are random with respect to some computable measure on 2^ω. However, in this more general setting the initial segment complexity of sequences that are random with respect to different computable measures can vary widely.

We study the various growth rates of proper sequences. In particular, we show the initial segment complexity of a proper sequence X is bounded from below by a computable function if and only if X is random with respect to some computable, continuous measure. We also identify a global computable lower bound for the initial segment complexity of all μ-random sequences for a computable, continuous measure μ. Furthermore we show that there are proper sequences with extremely slow-growing initial segment complexity, i.e., there is a proper sequence the initial segment complexity of which is infinitely often below every computable function, and even a proper sequence the initial segment complexity of which is dominated by all computable functions. Lastly, we prove various facts about the Turing degrees of such sequences and show that they are useful in the study of certain classes of pathological measures on 2^ω, namely diminutive measures and trivial measures.

[1] R. Hölzl and C. P. Porter, *Randomness for Computable Measures and Initial Segment Complexity*, ***ArXiv e-prints***, October 2015, 1510.07202.