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Short-run Dynamics and Long-run Effects of Demographic Change on Public Debt and the Budget

by

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Abstract*:

The German population is aging. Since fewer children are born and simultaneously life expectancy rises, demographic changes will lead to a double aging process. The paper analyzes the effects of demographic changes on national debt and the public budget by applying a cointegration analysis to global budget variables. Our procedure which covers the period between 1950 and 1990 completes prevailing projections, emphasizing that low vital rates are not the basic problem but the long-run trend of decreasing fertility and mortality rates. The estimation results of several error-correction models show that in the long-run an increase in the old age dependency ratio, and a decrease in the reproduction rate will lead to higher public expenditures. As regards public revenue, the results are ambivalent. The change in the age structure results in higher tax revenue, whereas the decline of population has the opposite effect. Furthermore, we find empirical evidence that aging increases the debt ratio. This development counters current efforts to reduce the public debt share of production potential and the tax load ratio. As a consequence, more action is needed to improve Germany's competiveness as a business location. Compared to these long-run effects, the short-run dynamics is only of minor importance. Significant parameter estimates can be found mainly in the model for the social security contribution rate.

1. Introduction

The German population is aging. At the beginning of this century, the age structure had a rectangular shape with a broad base, similar to a pyramid. At present, the age structure is similar to a fir-tree and if current projections prove to be correct, the population will experience significant aging in the first few decades of the next century resulting in an age structure that will come rather close to the shape of a mushroom (cf. Sommer [1994]). The process of aging is due to low vital rates, i.e. low birth and death rates. Birth rates have already declined markedly from their high post war levels, and are projected to decline further. This pressure is exacerbated by the observed trend to an increasing life expectancy. The effects of both processes sum up to what is called the "double aging process" (Börsch-Supan [1991], 107)[1]. The aging of the German population is particularly dramatic compared to other industrial countries. Therefore, an interesting question is whether the aging process can be stopped or even reversed by international migration which is the third central determinant of population growth, in addition to births and deaths. However, the rejuvenation effect of immigration depends on several conditions, i.e. the immigrants must be in for the greatest part of their births and their higher fertility rate should not adjust to the lower level of the native population. Simulation results show that mass immigration can fully offset population aging after a period of approximately 300 years (cf. Dinkel [1989], 310). However it would be wrong to conclude that immigration cannot help to alleviate the burden of aging. Calculations of Börsch-Supan ([1995], 47 p.) show that a liberal migration policy can help to stable population and to reduce the strain on the social security system. The impact of immigration on the social security burden, however, depends on the labor market absorption of the migrants. In the light of the present high unemployment figure, this assumption is rather controversial. Nevertheless, the aging process of the German population is undisputable, even if there is some uncertainty about its specific dimension.

In the discussion of the double aging process, it is often neglected that aging is not caused by low levels of the fertility or mortality rates (cf. Ulrich [1995], 23). Independent of the level of these rates, it can be shown that the population has a constant age

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[1] Births enter a population at age zero and hence always reduce the mean age. A rejuvenate effect also occurs if the deaths and the emigrants have a higher age than the mean age or if the immigrants are relatively young.

structure as long as the demographic parameters are stable over a longer period. Populations with low fertility or mortality rates are not necessarily aging populations. Even in this case, an invariant distribution of fertility and mortality rates will produce a constant age structure in the long-run that is independent from the initial age structure and is determined solely by the prevailing fertility and mortality regime.[2] From this point of view, it is not the low level of the current fertility and mortality rates that is causing concern but their decreasing trend in the long-run.

By means of cointegration analysis, the paper looks at the effects of an aging population on specific budget variables. In contrast to prevailing forecasts of the budgetary consequences of the future demographic development, our analysis concentrates on the development over the last 40 years.[3] Based on time series realizations of demographic and budgetary variables since 1952, our analysis emphasizes that aging is not caused by the current low level of the vital rates but is a phenomenon that has begun several decades ago. In this sense our analysis adds new insights to prevailing forecasts. Analyzing the aging issue with cointegration theory has the advantage, that the two central demographic variables, old-age-dependency ratio and net reproduction rate, enter the model simultaneously.[4] The conventional analysis of demographic change in a neoclassical model concentrates on the population growth rate and ignores the changing age pattern, which is often viewed as the more challenging problem (cf. Bös and Weizsäcker [1989], 346). Furthermore, our analysis allows to differentiate between short-run dynamics and long-run steady state effects.

Section 2 gives an overview over selected demographic and budgetary variables since 1952. The following section looks at cointegration theory and its relevance for population aging. Section 4 presents the results of our analysis. The paper concludes with some summarizing reflections.

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[2] This holds for a closed economy, i.e an economy without migration. The reason are the two ergodic theorems of population dynamics. In the long-run the prevailing fertility and mortality rates determine the development of population, whereas the present demographic structure is irrelevant (cf. Dinkel [1989], 128).
[3] Cf. Färber [1988], Felderer [1990], Wolfe [1993], and Boll [1994].
[4] We have already pointed to the importance of net immigration. Nevertheless, migration is not further considered. The migration time series has another order of integration. For details see section 3.

2. Demographic and budgetary time series

The paper examines short-run and long-run relations between demographic and budgetary variables. Therefore, we distinguish lower population growth, measured by the net reproduction rate (NRR), from population aging, measured by the old-age-dependency ratio (DR). The net reproduction rate informs about the extent to which a generation of daughters will replace their mothers and hence shows whether the population grows or shrinks. A net reproduction rate equal to one is necessary to re-place the population. In order to come to the replacement level, a woman has, on average, to bear more than one daughter, because the children may die before their own reproduction age.[5] For given mortality and net immigration rates, a lower fertility always implies an increase in the mean age of population (cf. Coale [1972], 117 p., Keyfitz [1985], 243 p.). Figure 1 shows the time series of the net reproduction rate between 1952 and 1989. The analysis concentrates on the situation before reunifica-tion. Basically, reunification has not reversed but only slowed down the process of population aging. Most population forecasts assume that the population of the new Länder takes up West German reproductive behavior in a medium term perspective (cf. Sommer [1994] 498 p.).

Up to the mid-sixties, the net reproduction rate run up to 1.2. After his relative maxi-mum, the NRR falls to a level of 0.6. After 1985, there is a slight recovery. As already mentioned, a net reproduction rate of one is necessary for replacement. Only in the interval between 1955 and 1970 we find a value greater than one. In all other years, Germany fails to reach the replacement level. After the end of the babyboom years, the net reproduction rate falls dramatically, having two local minima between 1978 and 1985. In 1990, the net reproduction rate is 0.67. This value is 33 percent below the required level for replacement and implies in the long-run that the population de-creases. Compared to the net reproduction rate, the old-age-dependency ratio in-forms about changes of the population age structure, which is basically determined by an increasing life expectancy. Population aging means an increase in the relative number of older persons, implying a shift towards the higher age groups (cf. Ulrich

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[5] In contrast to the net reproduction rate the aggregate birth rate doesn't consider the effect of mortal-ity during the period of fertility. It informs about how many children 1000 women would have to bear over their lifes, when the fertility rate is constant and no woman would die before reproduction. Taking into account the births of boys as well, the net reproduction rate needs approximately 2.2 births per woman (cf. Bretz [1986], 236 p.).

[1995], 22). It is best captured by the old-age-dependency ratio (DR), the ratio of eld-erly persons to non-elderly reproductive adults.[6] Aging occurs, if the dependency ra-tio increases. Figure 2 shows the time path of the old-age-dependency ratio since 1952.[7] Between 1952 and 1975, the dependency ratio increased steadily and reached a local maximum at 45 percent. After this period, the baby-boom generation broadened the population base and the dependency ratio began to decrease. In the eighties, the dependency ratio levels out at 42 percent which is equal to an increase of 40 percent since 1952. Compared to this development in the past, the expected increase is almost twice as high. According to the medium variant of population fore-cast of the German Federal Statistical Office, the dependency ratio will increase to 75 percent in the year 2040.

In addition to the demographic variables net reproduction rate and old-age-dependency ratio, five global budgetary variables enter our analysis (all measured relative to GDP): public expenditure ratio (PER), public debt ratio (PDR), tax load ra-tio (TLR), total contribution rate (TCR) and social contribution rate (SCR).[8]

The expenditure ratio is the sum of public expenditure (federal government, states, and communities) relative to GDP.[9] This ratio has a strong positive trend between 1952 and 1989. The time series reflects the main economic and fiscal decisions in Germany in the last four decades. High expenditure ratios can be found for the years of the oil crisis in the mid-seventies and at the beginning of the eighties. After 1982, the consolidation efforts of the new government and high economic growth rates re-sulted in a decline towards the 30 percent level. Parallely to the development of the expenditure ratio, the debt ratio increased significantly since 1952. In the past four decades, the debt ratio rose from 18 to 42 percent. Hereby, the debt ratio showed a relatively smooth development up to 1975. Afterwards, the oil crisis caused an explo-sion. At the end of the eighties, the debt ratio stabilized around 42 percent. The next variable, the tax load ratio has also a positive trend, however, the development is relatively uniform and stable. The main reason for the time pattern of

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[6] The old-age-dependency ratio is defined as ratio of persons aged 60 and above to the number of persons between 25 and 59 years. We have chosen the year 25 as the lower bound because the peri-ods of qualification have steadily increased over the past decades.
[7] Due to differencing, the first two years are lost.
[8] According to STATIS BUND and the National Accounts (cf. Statistisches Bundesamt [1991]).
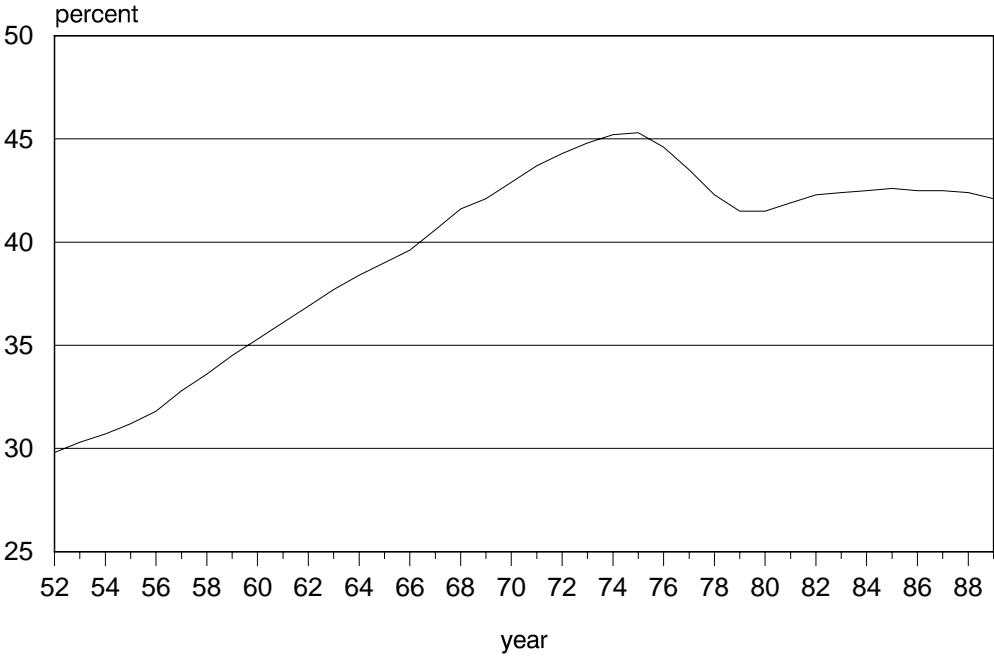[9] For details see Ulrich ([1995], 140 p.).

Figure 1: **Net reproduction rate (NRR) 1952-1989**

Source: Compiled and calculated from Statistisches Bundesamt [1994b].

Figure 2: **Old-age-dependency ratio (DR) 1952-1989**

Source: Compiled and calculated from Statistisches Bundesamt [1994b].

this variable is that the elasticity of tax revenue is close to one for the whole period. The total contribution rate is the sum of tax load ratio and social contribution rate. Since the tax load ratio developed relatively stable, the increase of the total contribution rate is caused by the increase of the social contribution rate. Within four decades, the total contribution rate has grown from 32 percent to over 42 percent. Here, the increase of the social contribution rate from 8 percent in 1952 to 17 percent in 1989 is the central component of this rise.

3. Cointegration and error-correction model
3.1 Cointegration Theory

Cointegration is closely related to the behavior of economic time series. Many time series are non-stationary, possessing a stochastic trend. Nevertheless, the variables are stable in the sense that they are „trending" together, so that a special linear combination of these variables is stationary. Cointegration means that there exists a linear combination between the variables, which is stationary, so that the long-run relation between cointegrated variables can be estimated consistently. In that case, an error-correction model can be formulated, that discriminates between short-run dynamics and long-run equilibrium.[10]

More precisely, a time series is called integrated of order one if it is non-stationary but the first difference is stationary. This is equivalent to the occurence of a unit root in the auto-regressive representation of the time series. Such a unit root implies an infinite summation (integration) of past error terms with non-decreasing weights. As a consequence, forecasts of I(1)-processes are uncertain because the variance increases without limits.[11] Applying standard estimation techniques to such variables conflicts with the stationary conditions of the classical regression model and may lead to spurious regression if estimation is carried out in the levels of such vari-

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[10] The aging issue of unemployment is studied by means of cointegration in Zimmermann ([1991], 177 p.). Welzel ([1994], 49 p.) performs causality tests between public expenditure and public revenue. Some macroeconomic implications can be found in Erbsland and Ulrich ([1992], 534 p.).

[11] A simple example for a nonstationary process with a variance function, that increases proportionate to the sample size, is the random walk: $x_t = x_{t-1} + \varepsilon_t$, where $\mathrm{var}(x_t) = \sigma_\varepsilon^2 \cdot t$ and ε_t is a stationary white noise process. In this case, differentiation leads to stationary: $\Delta x_t = x_t - x_{t-1} = \varepsilon_t$.

ables.[12] Solving this problem by going on to the first differences of the variables, it is important to realize that thereby the character of the error process is changed, and furthermore the relationship between the original variables will be lost. Cointegration determines the necessary conditions under which regressions in the levels of non-stationary variables lead to consistent estimates (see Granger [1986], Engle and Granger [1987]).

Let x_t be a vector, whose components x_{1t}, x_{2t} and x_{3t} are all integrated of order one. The variables x_{1t}, x_{2t} and x_{3t} are cointegrated if there exists (at least) one parameter vector $\lambda \neq 0$, such that

(1) $z_t = \lambda' x_t \sim I(0)$

where λ is the cointegration vector. Cointegration implies that the three I(1)-variables x_{1t}, x_{2t} and x_{3t} must be able to explain an I(0)-variable. This is not obvious because linear combinations of I(1)-variables are normaly again integrated of order one (see Wolters [1990], 159). From an economic point of view, this is of particular interest because if cointegration holds, the relationship between the original variables x_{it} (i = 1, 2, 3) is stable in the long-run since the deviation z_t from the equilibrium $\lambda' x_t = 0$ is an I(0)-variable. If one is not only interested in long-run equilibrium but also in short-run dynamics towards equilibrium, the following dynamic regression may be fitted successfully , which can be reparameterized as an error-correction model:

(2)
$$x_{1t} = a_0 + a_1 x_{1,t-1} + a_2 x_{1,t-2} + b_0 x_{2t} + b_1 x_{2,t-1} + b_2 x_{2,t-2}$$
$$c_0 x_{3t} + c_1 x_{3,t-1} + c_2 x_{3,t-2} + \varepsilon_{1t}$$

where ε_{1t} is a white noise error term.[13] The corresponding error-correction model is given by:

(3)
$$\Delta x_{1t} = \alpha_0 + \alpha_1 \Delta x_{1,t-1} + \beta_0 \Delta x_{2t} + + \beta_1 \Delta x_{2t-1} + \gamma_0 \Delta x_{3t} + \gamma_1 \Delta x_{3t-1}$$
$$\delta_1 (x_{1,t-1} - \lambda_2 x_{2,t-1} - \lambda_3 x_{3,t-1}) + \varepsilon_{1t} .$$

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[12] Spurious regressions results from the fact that time series with trend have an increasing variance, i.e. the auto-correlation coefficient of the residuals tends against one (see Granger and Newbold [1974]).
[13] The lag order is chosen in such that ε_{1t} is a white noise process, which by assumption is given by a lag order of 2.

with

$$\alpha_0 = a_0, \qquad \beta_1 = -b_2, \qquad \delta_1 = -(1 - a_1 - a_2),$$

$$\alpha_1 = -a_2, \qquad \gamma_0 = c_0, \qquad \lambda_2 = -\frac{(b_0 + b_1 + b_2)}{\delta_1},$$

$$\beta_0 = b_0, \qquad \gamma_1 = -c_2, \qquad \lambda_3 = -\frac{(c_0 + c_1 + c_{2)}}{\delta_1}.$$

In equation (3) the long-run multipliers (λ_2, λ_3) appear directly as parameters of the error-correction model.

3.2 Econometric method

3.2.1 Estimation of a vector auto-regressive process

The estimation follows directly equation (2). The basic model of cointegration theory is a p-dimensional vector auto-regressive (VAR)-process (see Johansen [1988], 232, Johansen and Juselius [1990], 170):

$$(4) \qquad x_t = A_1 x_{t-1} + A_2 x_{t-2} + ... + A_k x_{t-k} + c + \varepsilon_t, \qquad t = 1, ..., T,$$

where x_t is a ($p \times 1$) vector of stochastic variables, whereas the vectors x_{t-k}, ..., x_0 are predetermined, c is a constant term and the error terms ε_t are independently and identically distributed. In the following, it is assumed that the vector x_t is integrated of order one ($x_t \sim I(1)$). Similar to equation (2), equation (4) can be transformed such that Δx_t is explained by both lagged differences and lagged original variables (see Lütkepohl [1991], 356).

$$(5) \qquad \Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{k-1} \Delta x_{t-k+1} + c + \varepsilon_t, \qquad t = 1, ..., T.$$

Relation (5) is the error-correction representation of the VAR-process in equation (4). If in (4) $x_t \sim I(1)$ and hence is non-stationary, the matrix Π in (5) cannot have full rank. From $x_t \sim I(1)$ it follows that $\Delta x_t \sim I(0)$ and also ε_t is I(0) by assumption. If Π had full rank, it would be possible in (5) to solve for x_{t-1} and hence to explain a I(1)-variable by a finite number of I(0)-variables. If in the VAR-model of equation (4), r roots are equal to one, the Π matrix in equation (5) has r roots equal to zero[14]. Consequently, the

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[14] Adding x_{t-1} on both side of (5) leads to:

hypothesis of a reduced rank of the Π-matrix is equivalent to the hypothesis of coin-tegration. The null-hypothesis of r cointegration vectors can then be written as:

(6) $H_0(r) : \Pi = \alpha \lambda'$,

To obtain stationary variables for the estimation, we have to drop the zero roots in Π. In (6) we consider only the nonzero roots included in the transformation $\alpha \lambda'$. Hence, the term Πx_{t-1} in equation (5) can be replaced by the r canonical variables $\alpha \lambda' x_{t-1}$ or αz_{t-1}. The columns of λ are the r cointegrating vectors, which can be used to obtain stationary variables.

3.2.2 The method of Johansen

To estimate the cointegrating vectors λ we apply attechnique proposed by Johansen (Johansen [1988], Johansen [1995], Johansen and Juselius [1990]). The maximum likelihood estimation of model (5) is equivalent to the solution an eigenvalue problem and gives estimaters for the cointegrating vectors $\hat{\lambda}$ and the parameters $\hat{\alpha}$. Hereby, the estimators $\hat{\lambda}$ are the r largest characteristic roots of $\square$ (see Johansen and Juselius [1990], 177). Compared to the method of Engle and Granger or Stock, the main advantage of this method is, that is does not require to assume that just one cointegrating relationship exists.[15] The problem of determining the number r of coin-tegration vectors can be solved by a likelihood ratio test. A comparison of the maxi-mum likelihood function under the hypothesis of r cointegration vectors with the unre-stricted maximum yields the following ratio:

(7) $LR = \prod\limits_{i=r+1}^{P} (1 - \hat{\mu}_i)^{-\frac{T}{2}}$.

If the null-hypothesis holds, the p-r smallest eigenvalues in (7) are close to zero since

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$x_t = A x_{t-1} \Gamma_1 \Delta x_{t-1} + \ldots + \Gamma_{k-1} + c + \varepsilon_t$. The model is non-stationary if some roots of A are equal to one. Since $\Pi = (A-I)$, the zero roots correspond to the unit roots of A.

[15] The methods of Engle and Granger [1987] or Stock [1987] require the assumption that there is ex-actly one cointegrating relation.

the rank of Π is equal to the number of nonzero eigenvalues. Hence, the likelihood ratio (7) tends to one. Johansen ([1988] and together with Juselius [1990]) has tabulated the distribution of the following test statistics, which is called the trace test:

$$(8) \qquad -2\ln LR = -T \sum_{i=r+1}^{P} \ln(1-\hat{\mu}_i).$$

The null-hypothesis of r cointegration vectors will be rejected if the p-r smallest roots are large enough to make (8) greater than the critical value. An alternative test statistics (eigenvalue statistic) compares the two greatest eigenvalues and hence the hypothesis H_0(r-1) against H_1(r), i.e. the model has r-1 or r cointegrating vectors, respectively:

$$(9) \qquad -2\ln LR = -T \ln(1-\hat{\mu}_r).$$

Having estimated the cointegrating vectors, they can be used to build the stationary variables z_t of the error-correction model (5), whose coefficients $\Gamma_1,...,\Gamma_{k-1}$ can then be estimated consistently.

4. Empirical results

4.1 Cointegration Tests

To deal with the possible non-stationarity of the time series, the first step is to find the number of unit roots in the auto-regressive process. Therefore, we determine the order of integration of the time series. The most commonly used test is the Augmented-Dickey-Fuller (ADF)-test (see Fuller [1976], Dickey and Fuller [1979], Said and Dickey [1984]), which is based on the following regression:

$$(10) \qquad \Delta X_t = \alpha_0 + \alpha_1 X_{t-1} + \sum_{i=1}^{p} \alpha_{2i}\Delta X_{t-i} + \gamma_0 T + u_t,$$

The normal t-value can be used to test the hypothesis $\alpha_1 = 0$ in (10). Nevertheless, its distribution is non standard and the critical values are derived from monte carlo

methods. The ADF-test may be applied with ($\gamma = 0$) and without ($\gamma \neq 0$) a deterministic trend. As a result, we find that all seven time-series are integrated of order one and hence I(1)-variables.[16]

Having determined the order of integration, the next step is testing for cointegration. In our specific case, cointegration means, that there is a long-run equilibrium between each budget variable and the two demographic variables net reproduction rate and old-age-dependency ratio.[17] Therefore, we estimate five models: the public expenditure model, the public revenue model, the public debt model, the social contribution rate model, and the total contribution rate model. Each model consists of three time series, i.e. one budget variable and the two demographic variables net reproduction rate and old-age-dependency ratio.

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[16] A detailed description of the test-results can be found in Ulrich ([1995], 157).

[17] A problem may be that rates are bounded between zero and one and hence cannot have a stochastic or deterministic trend. However, this is not true for rates, where the numerator is not a part of the denominator, i.e. the debt ratio. Nevertheless, even in the case of genuine rates, there exists a transformation that solves the boundary problem without modifying the cointegrating relationship (Allen [1994], 15). In an paper on aging and unemployment rates, Zimmermann ([1991], 188) uses the notion of long-run only in a local sense.

Table 1:   **Eigenvalues and eigenvectors of the five models**

Variable	Model: PER, NRR ,DR		
	Eigenvalues		
	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$
	0.4244	0.2066	0.0940.
	Eigenvectors		
	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$
PER	1.000	1.000	1.000
DR	-0.993	-0.235	0.198
NRR	2.595	6.713	124.185
Variable	Model: PDR, NRR, DR		
	Eigenvalues		
	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$
	0.6648	0.13807	0.0269
	Eigenvectors		
	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$
PDR	1.000	1.000	1.000
DR	-3.151	1.193	0.111
NRR	34.390	82.095	-5.153
Variable	Model: TLR, NRR, DR		
	Eigenvalues		
	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$
	0.4698	0.3063	0.0994
	Eigenvectors		
	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$
TLR	1.000	1.000	1.000
DR	-0.174	0.234	0.142
NRR	-0.192	0.844	19.294
Variable	Model: TCR, NRR,DR		
	Eigenvalues		
	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$
	0.4542	0.1728	0.0943
	Eigenvectors		
	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$
TCR	1.000	1.000	1.000
DR	-0.841	-0.257	-0.901
NRR	7.707	10.900	-55.969
Variable	Model: SCR, NRR,DR		
	Eigenvalues		
	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$
	0.5794	0.2408	0.0679
	Eigenvectors		
	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$
SCR	1.000	1.000	1.000
DR	-0.546	-0.007	-0.445
NRR	9.118	12.974	-13.131

Source: own calculations using CATS in RATS.

Table 1 shows the eigenvalues and the normalized eigenvectors of the five models. In order to determine the number of cointegrating vectors, the eigenvalues of table 1 are inserted into the equations (8) and (9). The resulting test statistics are given in table 2. According to the trace statistic[18], the null-hypothesis of no cointegration between the budgetary and the demographic variables (r=0) is rejected at the 5 percent level of significance.[19] Having rejected the hypothesis of no cointegration, we next ask for the exact number of cointegrating vectors. Between three variables, all integrated of order one, there are up to two cointegrating relationships. The hypothesis of exactly one cointegrating vector cannot be rejected for the expenditure model, the debt model, the social contribution rate model, and the total contribution rate model (see table 2).[20] Only for the tax load ratio model, the hypothesis of one cointegration relation can be rejected. Here, the critical values from table 2 indicate that two cointegrating vectors occur.

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[18] For the problem of a small sample, the trace test is more robust against violations of the normal distribution assumption (see Cheung and Lai [1993], 322 p.).

[19] The critical values of the test statistics are taken from table B3 of Hansen and Juselius ([1995], 81) or from table 1 of Osterwald-Lenum ([1992], 468), respectively. The following example illustrates the calculations. Under the null-hypothesis of no cointegration (r = 0), the trace statistic for the public expenditure model is:

$$-36\sum_{i=1}^{3}\ln\left(1-\hat{\mu}_i\right) = -36\left[\ln(1-0.4244)+\ln(1-0.2066)+\ln(1-0.0940)\right] = 31.77 .$$

Since 31.77 is greater than the critical value 29.38 (Hansen and Juselius [1995], table B3) or 29.68 (Osterwald-Lenum [1993], table 1), the hypothesis of no cointegration can be rejected at the 5 percent level. Next, the trace statistic for the hypothesis of one cointegrating relation (r ≤ 1) is:

$$-36\sum_{j=r+1}^{3}\ln\left(1-\hat{\mu}_j\right) = -36\left[\ln(1-0.2066)+\ln(1-0.0940)\right] = 11.88 .$$

Here, the null hypothesis cannot be rejected (the critical values are 15.34 or 15.41). Taken together, the tests come to the result that exactly one cointegrating vector exists between the public expenditure ratio, the net reproduction rate and the old-age-dependency ratio.

[20] Both the trace test and the eigenvalue test are only asymptotically valid and hence a small sample problem may arise. Simulations of Reimers [1991] point to this problem. He concludes, that in small samples, the hypothesis of no cointegration is rejected too often. In our specific case, however, the application of his proposed corrections leads only to marginal modifications.

Table 2:   **Tests of cointegration**

Number of cointegrating vectors r	PER, NRR, DR	
	Trace statistic	eigenvalue statistic
r ≤ 2	3.55	3.55
r ≤ 1	11.88	8.33
r = 0	31.77[a)]	19.89[b)]
	PDR. NRR, DR	
	Trace statistic	eigenvalue statistic
r ≤ 2	0.98	0.98
r ≤ 1	6.33	5.35
r = 0	45.68[a)]	39.35[a)]
	TLR, NRR, DR	
	Trace statistic	eigenvalue statistic
r ≤ 2	3.77	3.77
r ≤ 1	16.93[a)]	13.17[a)]
r = 0	39.78[a)]	22.84[a)]
	TCR, NRR, DR	
	Trace statistic	eigenvalue statistic
r ≤ 2	3.57	3.57
r ≤ 1	10.40	6.83
r = 0	32.19[a)]	21.80[a)]
	SCR, NRR, DR	
	Trace statistic	eigenvalue statistic
r ≤ 2	2.53	2.53
r ≤ 1	12.45	9.92
r = 0	43.67[a)]	31.22[a)]

a) 5 percent level of significance.
b) 10 percent level of significance.
Source: Own calculations.

4.2 Weak exogeneity

Johansen´s estimation method allows for testing of exogeneity. For our analysis, a test for (weak) exogeneity between the demographic and the budgetary variables is of particular interest. Obviously, demographic conditions have an impact on public expenditure and public revenue. Public pensions or statutory health insurance are good examples. On the other hand, the decision to retire or the decision to have children depend also on public budget variables, i.e. the conditions of early retirement or the size of family subsidies. Our model allows to test, whether demographic variables are exogenous in the long-run, or whether there is an expected simultaneous relationship. Because exogeneity is only referred to the long-run steady state, we talk about weak exogeneity.

We start with the error-correction representation of equation (5). If the model in equation (5) has weakly exogenous variables, the vector $x_t = \left(x_{t1}, ..., x_{tp} \right)'$ can be parti-

tioned as follows (see Juselius [1991], 271, Harris [1994], 1233):

$$(11) \quad x_t = \left(y_t, z_t\right)',$$

with y_t as stochastic variables and z_t as the subset of weakly exogenous variables. Consider $k=2$ (order of the process) and $r=2$ (two cointegrating vectors) as in our model, equation (5) can be written:

$$(12) \quad \begin{bmatrix} \Delta y_t \\ \Delta z_t \end{bmatrix} = \Gamma_1 \begin{bmatrix} \Delta y_{t-1} \\ \Delta z_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} x_{t-1} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}.$$

If the parameters $\alpha_2 = \left(\alpha_{21}, \alpha_{22}\right) = 0$, z_t is (weakly) exogenous for the long-run parameters λ_i. In this case, the z-variables are of no relevance for the estimation of the cointegrating vectors. Here, the long-run equilibrium relationship $\lambda_j' x_{t-1}$ is not included in the i-th equation. Then, equation (12) can be conditioned on the marginal distribution of Δz_t, and the parameters α and λ can be estimated by the conditional model (see Johansen [1995], 122):

$$(13) \quad \Delta y_t = A_0 \Delta z_t + A_1 \Delta x_{t-1} + A \lambda' x_{t-1} + c + \varepsilon_t, \qquad t = 1, \dots, T.$$

Equation (13) shows that only the short-run parameters (A_0, A_1) are changed in the case of (weak) exogeneity but not the long-run parameters λ, which are used to form stationary or canonical variables. The test for (weakly) exogenous variables in the model is based on the following likelihood ratio statistic (Johansen [1995], 126):

$$(14) \quad -2 \ln L R = T \sum_{i=1}^{r} \ln \left\{ \left(1 - \tilde{\mu}_i\right) / \left(1 - \hat{\mu}_i\right) \right\},$$

where $\tilde{\mu}_i$ are the eigenvalues of the constrained model and $\hat{\mu}_i$ are the eigenvalues of the unconstrained model. The test statistics follows asymptotically a χ^2-distribution with $r(p-m)$ degrees of freedom (Johansen [1995], 126). We apply this procedure to test whether the old-age-dependency-ratio ($\alpha_2 = 0$), the net reproduction rate ($\alpha_3 = 0$) or both demographic variables together are exogenous. Table 3 shows the result of

the exoneneity tests.

Tabel 3: Test for (weak) exogeneity of the demographic variables[1]

Model	Hypothesis		
	DR exogenous $\alpha_2 = 0$	NRR exogenous $\alpha_3 = 0$	DR and NRR exogenous $\alpha_2 = \alpha_3 = 0$
PER, NRR, DR r = 1	1.31 (1)	3.66 (1)[a]	4.45 (2)
PDR, NRR, DR r = 1	4.03 (1)[b]	6.67 (1)[b]	11.18 (2)[b]
TLR,NRR, DR r = 2[2]	3.67 (2)	7.92 (2)[b]	15.10 (4)[b]
TCR, NRR, DR r = 1	4.16 (1)[b]	4.03 (1)[b]	7.99 (2)[b]
SCR, NRR, DR r = 1	1.74 (1)	3.33 (1)[a]	4.58 (2)

a) 10% level of significance.
b) 5% level of significance.
1) Degrees of freedom in brackets. The 10% level of significance with 1 degree of freedom is 2.71, with 2 degrees of freedom 4.61, and with 4 degrees of freedom 9.94. The figures for the 5%-level are 3.84, 5.99 and 11.14.
2) For two cointegrating vectors the restriction is $\alpha_{21} = 0$, $\alpha_{31} = 0$, i = 1, 2.
Source: Own calculations.

As expected, the results in table 3 indicate that the exogeneity assumption does not hold in all cases. It is interesting to notice the differences between the net reproduction rate and the dependency ratio. In most cases, the old-age-dependency ratio ($\alpha_2 = 0$) is (weakly) exogenous, while the net reproduction rate is endogenous ($\alpha_3 = 0$), having a feed back relationship to the global budget variables.

Since nearly all population forecasts give more weight to the dependency ratio in the next three to four decades, there will be an interesting difference between the past and the future demographic development and the resulting budgetary effects of demographic change in the long-run.

According to these results, we assume the net reproduction rate and the age-dependency ratio to be exogenous in the expenditure model and in the social contribution rate model. In the tax load ratio model, only the the old-age-dependency ratio is (weakly) exogenous. Finally, in the debt ratio model and in the total contribution rate model both variables are endogenous. In case of exogeneity, the cointegrating vectors are reestimated (table 4). The reestimated cointegrating relations are used to form the stationary variables z (table 5).

The cointegrating vector of the first model (Z1) shows that the double aging process, i.e. a decreasing net reproduction rate and an increasing old-age-dependency ratio, will rise public expenditure in the long-run. For the models 4 (TCR) and 5 (SCR), these changes imply that the financial strain on the social security and tax system will further increase. In the long-run, double aging worsens the situation of Germany as a business location because of the increasing indirect labor costs. As regards public debt, double aging implies an increase in the debt ratio (see table 5, model 2 (PDR)). Hence, the aging and shrinking German population puts further burden to future generations.

Table 4:   Estimated cointegrating vectors

	Cointegrating relations			
Model	No	$\alpha_2 = \alpha_3 = 0$	$\alpha_2 = 0$	$\alpha_3 = 0$
PER r = 1	1.0000 -0.993 2.595	1.000 -0.700 3.928	1.000 -0.941 1.738	1.000 -0.730 4.600
Model	No	$\alpha_2 = \alpha_3 = 0$	$\alpha_2 = 0$	$\alpha_3 = 0$
PDR r = 1	1.0000 -3.154 34.390	1.000 -2.458 38.660	1.000 -3.125 31.738	1.000 -2.590 40.654
Model	No	$\alpha_{2i} = \alpha_{3i} = 0$ i = 1, 2	$\alpha_{2i} = 0$ i = 1, 2	$\alpha_{3i} = 0$ i = 1, 2
TLR r = 2	1.000 1.000 -0.174 0.234 -0.192 0.844	1.000 1.000 -0.132 0.108 -0.363 12.164	1.000 1.000 -0.144 0.440 -0.464 4.298	1.000 1.000 -0.143 0.903 -0.039 -28.719
Model	No	$\alpha_2 = \alpha_3 = 0$	$\alpha_2 = 0$	$\alpha_3 = 0$
TCR r = 1	1.000 -0.841 7.707	1.000 -0.656 7.540	1.000 -0.896 6.451	1.000 -0.737 9.384
Model	Keine	$\alpha_2 = \alpha_3 = 0$	$\alpha_2 = 0$	$\alpha_3 = 0$
SCR r = 1	1.000 -0.546 9.118	1.000 -0.485 9.429	1.000 -0.531 8.950	1.000 -0.497 9.598

Source: Own calculations.

Table 5: Estimated long-run relationships

Model 1: Variables:PER, NRR, DR; Restrictions: r = 1, $\alpha_2 = \alpha_3 = 0$.   Cointegrating relation: Z1 = PER - 0,700 DR + 3,928 NRR
Model 2: Variables: PDR, NRR; DR; Restrictions: r = 1.   Cointegrating relation: Z2 = PDR -3,154 DR + 34,390 NRR
Model 3: Variables: TLR, NRR; DR; Restrictions: r = 2, $\alpha_{2i} = 0$, i = 1, 2.   Cointegrating relation 1: Z31 = TLR - 0,144 DR - 0,464 NRR   Cointegrating relation 2: Z32 = TLR + 0,440 DR + 4,298 NRR
Model 4: Variables: TCR, NRR; DR; Restrictions: r = 1.   Cointegration relation: Z4 = TCR - 0,841 DR + 7,707 NRR
Model 5: Variables: SCR, NRR; DR; Restrictions: r = 1, $\alpha_2 = \alpha_3 = 0$ Cointegration relation: Z5 = SCR - 0,485 DR + 9,429 NRR

Source: Own Calculations.

In the tax load ratio model we found two cointegrating vectors. The first long-run relationship (Z31) has the consequence that both an increasing dependency ratio and a rise in the net reproduction rate will rise public revenue and hence result in a higher tax load ratio. The second cointegrating vector has exactly the opposite implication. Consequently, there is no clear cut interpretation of the long-run effects. In the next section 4.3 we will see that the second cointegrating vector is insignificant in the error-correction model, implying that the relationship Z31 gives the correct long-run relationship between public revenue and demographics. Then, a shrinking population goes along with declining public revenue and the increasing share of older people rises public revenue.[21]

4.3 Error-Correction Model

From section 3 we know that in case of cointegration an error-correction model can be formulated that comines short-run dynamics and long-run equilibrium. The error-correction model combines both long-run relations and short-run dynamics. The short-run dynamics is the direct and immediate reaction of the budget to demographic changes, whereas the long-run equilibrium is the steady state relationship in the levels of the variables. Estimation follows equation (5). To obtain stationarity, Πx_{t-1} in equation (5) is replaced by the canonical cointegrating vectors. Table 6 contains

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[21] In a study for the Netherlands, Goudswaard and van de Kar ([1994], 58) found that for the period 1987 to 2010 the increasing share of elderly people will rise tax revenue by 27 percent.

the estimation results of the different error-correction models.[22]

All parameters of the cointegrating vectors (Z1 to Z5) are highly significant, indicating that there is a long-run relationship between demographic and budgetary variables. As regards the the error-correction model for the tax load ratio, only the first cointegrating vector (Z31) is significant. This explains the above interpretation that the second relationship is irrelevant. Concentrating on Z31, double aging has different and variable effects on the left hand side of the budget.

Turning to the estimation results of table 6, the most striking result is that short-run dynamics is only of minor importance compared to the long-run equilibrium relationship between demographic and budgetary variables. For example, short-run dynamics shows an immediate adjustment of both public debt and social security contributions to changes in the net reproduction rate (ΔNRR(-1)). More precisely, an increase in the fertilitiy rate rises both public debt and the contribution rate. A possible explanation is that an increase in the net reproduction rate directly increases the number of dependent and hence subsidized young people, implying additional social expenditure. Additionally, we find that an increase in the dependency ratio (ΔDR) rises public expenditure in total and social expenditure in particular. Here, aging causes additional burden even in the short-run.

The conclusions from the estimated error-correction models are, that in the past demographic change had mainly brought about additional burden on the right-hand side of the budget. The double aging process of the next three to four decades will lead to additional pressure on public expenditure and public debt. Especially the old-age-dependency ratio will rise quickly because the number of the elderly in the numerator increases due to the rising life expectancy while at the same time the number of the younger people in the denominator shrinks due to the declining fertility. According to the pevailing pay-as-you-go systems, financing of public goods and social services will be more difficult. This development counters current efforts

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[22] Because all variables of the error-correction model are stationary, the t-value can be used to test the significance of the parameters.

Table 6:   **Estimation of different budget models with cointegration**

Explanatory variables	Dependent variables				
	ΔPER	ΔPDR	ΔTLR	ΔTCR	ΔSCR
Constant	1.775[a] (4.238)	-4.741[a] (-3.631)	15.253[a] (4.496)	2.634[a] (3.728)	0.500[a] (6.186)
Z1(-1)	-0.378[a] (-4.391)	----------	----------	----------	----------
Z2(-1)	----------	-0.073[a] (-4.043)	----------	----------	----------
Z31(-1)	----------	----------	-0.724[a] (-5.208)	----------	----------
Z32(-1)	----------	----------	-0.038 (-0.811)	----------	----------
Z4(-1)	----------	----------	----------	-0.240[a] (-3.616)	----------
Z5(-1)	----------	----------	----------	----------	-0.287[a] (-6.281)
ΔPER(-1)	0.234c) (1.722)	----------	----------	----------	----------
ΔPDR(-1)	----------	0.353[a] (3.162)	----------	----------	----------
ΔTLR(-1)	----------	----------	0.194 (1.471)	----------	----------
ΔTCR(-1)	----------	----------	----------	-0.044 (-0.323)	----------
ΔSCR(-1)	----------	----------	----------	----------	0.238[b] (2.093)
ΔDR	1.187[a] (3.016)	----------[2]	-0.452 (-1.540)	----------[2]	0.546[a] (4.189)
ΔNRR	5.081 (1.460)	----------[2]	----------[2]	----------[2]	-0.575 (-0.508)
ΔDR(-1)	-1.022[b] (-2.459)	0.291 (0.705)	-0.255 (-0.747)	0.278[b] (1.709)	-0.194 (-1.430)
ΔNRR(-1)	2.415 (0.643)	18.771 (2.730)	-4.238[c] (-1.699)	3.387 (0.995)	5.335[a] (3.816)
R^2	0.443	0.500	0.514	0.368	0.683
$\overline{R}^2$	0.328	0.435	0.413	0.287	0.618
Q(9)	9.697	6.987	5.431	4.300	8.6856
SK	0.600	0.085	-0.471	-0.189	-0.3879
KU	0.807	-0.171	0.118	-0.020	0.2529

a) 1% level of significance.
b) 5% level of significance.
c) 10% level of significance.

1) t-values in brackets. R² ist the coefficient of multiple determination and $\overline{R}^2$ is the adjusted coefficient. Q(9) is the Q-statistic with 9 degrees of freedom of Ljung Box. The statistic is not significant in any of the error-correction models. SK is the estimated skewness and KU the estimated kurtosis of the residuals. For all models the hypothesis of normal distributed residuals cannot be rejected.
2) DR and NRR are endogenous and hence are excluded as right-hand-side variables.
Source: Own calculations.

to reduce the public-debt share of production potential and the social contribution load ratio. As regards the revenue side, we found a positive impact of demographics on the tax load ratio. Here, it is difficult to forecast the future development. On the one hand, total population is decreasing and on the other hand, there will be a sig-

nificant change in the tax structure since the ratio of consumers to working individuals increases.

5. Summary

The expected change in the size and the age structure of the German population is dramatic. The Federal Statistical Office forecasts a decline from 82 millions to 72 millions over the next four decades. The paper analyzes the effects of the double aging process on the public budget. On the revenue side of the budget, there will be changes of tax payment and tax structure since the relation between working population and consumers will be altered. On the right-hand-side of the budget, size and structure of public expenditure will change. Here the increasing number of dependent people is a real danger for the social security system because the prevailing pay-as-you-go system works only with a stable population.

The paper analyzes the effects of demographic change on selected budget variables by applying cointegration theory. Our procedure, which covers the period between 1950 and 1990, completes prevailing projections of demographic change, emphazising that low vital rates are not the basic problem of aging but the long-run trend of decreasing fertility and mortality rates.

Two demographic variables, the net reproduction rate and the old-age-dependency ratio enter the analysis together with five budgetary variables (all measured in percent of GNP): expenditure ratio model, debt ratio model, tax load ratio model, total contribution rate model and social contribution rate model. The cointegration analysis shows the importance of the long-run relationship between budgetary and demographic variables. The long-run equilibrium is highly signifcant in all five models. In the long-run, the increase of the old-age-dependency ratio and the decrease in the net reproduction rate will lead to higher public expenditure. As regards the social and the total contribution rate, we find similar effects. For the debt model, demographic change will lead to a higher public debt. On the left-hand side of the budget, the estimated cointegrating vector shows a positive relation between tax load ratio and the two demographic variables. An aging but initially still growing population produces higher tax revenue. Compared to these long-run adjustment effects, short-run dy-

namics is less important. Here, we found a short-run adjustment mainly in the social contribution rate model.

The future demographic development differs substantially from the analyzed development in the past. Double aging will lead to a sharp increase of the social contribution rate. Together with income taxes, the total payroll tax burden will exeed any reasonable limit, impeding the willigness to work and supporting the exodus from the market into the shadow economy. However, despite of this rather pessimistic view, it should be emphasized, that the government has many instruments to absorb the shocks of demographic change and to guide the economy in a better direction. Indeed, a liberal immigration policy, an increase in labor hours, in retirement age, in female labor force participation, a reduction of periods of training or an active family policy can help to reduce or even to offset the macroeconomic effects of population aging. Furthermore instead of raising employment in an aging country capital can be brought to countries with less scarcity of labor and hence to allocate capital and labor to their most efficient allocation. However, even if we are able to solve the aging problem, we should not be too optimistic because of the danger that policy makers choose the wrong instruments. Here, the implementation of a pay-as-you-go system for long-term care in Germany is a good example.

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