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Optimal fiscal policy, uncertainty, and growth

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Abstract

This paper analyzes the macroeconomic effects of fiscal policy in a stochastic endogenous growth model. Due to externalities in human capital accumulation, the market allocation is inefficient, thereby justifying government intervention. The uncertainty stemming from technological disturbances affects the growth rate, which can be explained by precautionary motives of risk averse agents. Fiscal policy means consist of a consumption tax, investment subsidies, and bonds. We obtain counter-acting growth effects of investment subsidies, which are differentiated with respect to deterministic and stochastic capital income components. The policy implications from the deterministic model are substantially extended in the stochastic context. A general rule for a welfare maximizing policy is derived, which is represented by a continuum of alternative tax-transfer-schemes. We discuss three benchmark cases, which crucially differ with respect to their implications regarding the size of the government expenditure share.

JEL classification: D8; D9; E6; O4
Keywords: Endogenous growth; Optimal taxation; Uncertainty; Government debt

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1. Introduction

This paper is concerned with the macroeconomic effects of public tax-transfer schemes within the context of a stochastic general equilibrium model. By extending the widely used Romer (1986) learning-by-doing framework with productivity shocks, we illustrate that the introduction of uncertainty goes along with interesting implications for public intervention that may differ significantly from those derived for the standard model. We derive conditions under which the optimal policy design even is reversed in a risky environment when compared to the deterministic setting, and find that this result can be attributed to two important factors: the ambiguous impact of technological uncertainty on growth and the partially counteracting effects of fiscal policy under risk.

The paper addresses two major issues. First, we focus on the impact of unpredictable technology shocks on long-run growth. This area has lately gained new attraction with an increasing amount of contributions; see Obstfeld (1994), Cazzavillan (1996) and Smith (1996a). The extension of modern endogenous growth theory with random disturbances suggests itself. It especially allows for a combined analysis of growth and business cycle phenomena; see Jones et al. (1999), Collard (1999) and Fatás (2000). In general, the argument stems from the endogeneity of the growth rate, dependent on the fundamentals of the economy and now extended by volatility as another key factor. For reasons which we will discuss below, the results from the theoretical and empirical work can be summarized in the simple phrase 'uncertainty matters', and the model presented here will provide no exception to this rule.

Second, our paper adds to the extensive recent literature concerned with the effects of fiscal policy on macroeconomic performance in stochastic growth models; see Turnovsky (1993, 1995, 1999a,b, 2000), Smith (1996b), and Corsetti (1997). We derive a general condition for an optimal policy and discuss alternative combinations of fiscal instruments that suffice the requirement of maximum welfare. Our paper differs from the ones just cited to one major respect. These contributions suffer from the shortcoming that usually the technology is restricted to reproducible factors of production. To be more precise, either capital is the single factor of production or the intertemporal flow of labor incomes is captured as lifetime human wealth, thus being regarded as 'quasi' accumulable. With the only exception of Clemens and Soretz (1997, 1999), the authors do not explicitly consider stochastic labor incomes, thereby ignoring the impact of different sources of income on growth. We assume both, capital and income risk, and point out the importance of distinct reproducible and non-reproducible income sources for the analysis of macroeconomic growth and optimal fiscal policy.

We investigate a Ramsey-type economy where the growth rate of the economy is determined by the factors that affect individual savings. In this context it is only natural to consider the impact of technological shocks on capital accumulation. We draw from Jones et al. (1999), who only recently brought the argument to attention again that short-run fluctuations may have long-lasting effects on macroeconomic trend variables. At a theoretical level, this can be explained by precautionary motives that may appear if agents are sufficiently risk averse. The analysis of precautionary saving traces back to Leland (1968) and Sandmo (1970), and is empirically supported by the work of Hubbard et al. (1994). In this context, uncertainty is a relevant factor of the intertemporal savings decision of a risk averse individual. Precautionary savings are then characterized by a higher propensity to save when compared to a riskless environment, and can be regarded as self-insurance against future income risk. We demonstrate that in a learning-by-doing growth model, this risk-induced accumulation may even lead to excessive, that is suboptimally high growth.

The aggregate income risk assumed in this paper affects the macroeconomic equilibrium in various ways and by this is a substantial determinant for the efficacy of any tax-transfer policy. The argument of Domar and Musgrave (1944) and Stiglitz (1969) will be important for our analysis, who pointed out that taxation of returns to risky assets may actually increase the demand for those assets. This argument can be carried over to investment subsidies which affect the mean as well as the volatility of future capital incomes. We distinguish between two contrary effects: A rise in expected return is an incentive to increase capital accumulation whereas a higher riskiness discourages growth.

The agents of the Romer (1986) model neglect the external effect of human capital accumulation. The private return to physical capital falls short of the social return. Usually, a competitive equilibrium is characterized by a suboptimally high propensity to consume and suboptimally low savings compared to the efficient path. But additionally, now we have to account for capital volatility which also is underestimated and causes the agents to increase savings towards a suboptimally high level. From this we conclude that it is by no means obvious that the standard results for an optimal policy continue to hold. An optimal policy might even be characterized by growth depressing means.

The standard approach for this class of model is that growth should be enhanced with a fiscal tax-transfer scheme that either discourages consumption via a consumption tax or that works through investment incentives via an investment-tax credit or a production subsidy respectively. We in general follow this line of argument by assuming that government grants a production subsidy that is mainly financed out of revenues from a consumption tax. We do not impose a balanced budget in every period, thereby allowing the government to borrow and lend on the financial markets. In addition to the intertemporal allocation of consumption and saving, the household then has to decide on the optimal structure of his portfolio of assets.

The policy parameters, the consumption tax rate as well as the production subsidy, affect the optimal portfolio choice and the welfare maximizing propensity to consume over time. But recalling the volatility argument from above, we demonstrate that the policy implications known from the deterministic setting not necessarily extend to the stochastic model. We show that the welfare effects from a change in optimal consumption and portfolio choice can be summed up in the growth effects. Although the welfare effect of a specific tax-transfer policy is ambiguous, it is possible to derive conditions for optimal policies. We demonstrate that these policies differ to the extent the underlying fiscal instruments are available to attain the optimal allocation.

The paper is organized as follows. Section 2 develops the model and presents the results from individual optimization. In Section 3, the macroeconomic equilibrium is derived. We focus on the specific incidence of tax and transfer parameters on the equilibrium value of the expected growth rate. Section 4 is devoted to the question of optimal policies. We discuss several benchmark cases. Section 5 briefly summarizes the results.

2. The model

The representative firm produces a homogeneous good according to the stochastic Cobb-Douglas technology

$$\mathrm{d}Y(t) = \gamma K(t)^{\alpha}[L(t)A(t)]^{1-\alpha}[\mathrm{d}t + \sigma\,\mathrm{d}y(t)], \quad \alpha \in (0,1), \ \gamma > 0. \tag{1}$$

The instantaneous output $\mathrm{d}Y(t)$ is subject to an economy-wide productivity shock. [1] $\mathrm{d}y(t)$ is a serially uncorrelated increment to a standard Wiener process with zero mean and variance $\mathrm{d}t$. The population is assumed to be constant. Labor $L(t)$ is supplied inelastically and normalized to unity. $K(t)$ is the stock of physical capital. The production function of the representative firm has constant returns to scale with respect to capital and labor. Due to the productivity shock, the returns from the two privately owned factors of production are stochastic. In terms of Sandmo (1970), the household is subject to a capital and an income risk. Following Romer (1986), $A(t)$ represents the stock of technical knowledge of the economy. It displays the characteristics of a public good and is enhanced by investment in physical capital. In equilibrium, $A(t)$ is equal to $K(t)$. On the aggregate level, the production function is linear in the reproducible factor, thereby inducing ongoing growth of per-capita incomes. At each instant of time the mean and variance of the conditional distribution of $\mathrm{d}Y$ depends on the existing capital stock. For this reason, current shocks will have long-lasting effects on the output process to the extent that they affect capital accumulation over time.

We specify a linear aggregate tax function of the following form:

$$\mathrm{d}T(t) = \tau C(t)\,\mathrm{d}t, \quad \tau \in [0,1], \tag{2}$$

where $\mathrm{d}T(t)$ denotes the flow of tax revenues, $C(t)$ is instantaneous consumption of time t and τ is a time-invariant tax rate. Because we assume labor to be inelastically supplied, there is no labor–leisure choice and the economic effects from a taxation of wage incomes do not differ from taxing consumption. Wage taxes then can be neglected without loss of generality.

The agents revenues out of physical capital are subsidized. In following Eaton (1981), we posit that the government is able to subsidize capital returns at separate, time-invariant rates, θ_d, θ_s, in order to distinguish between permanent and transitory capital incomes. This assumption captures the idea that investments are treated dif-

[1] $Y(t)$ denotes the cumulative production of time t.

ferently, depending on the associated risk. In the proceedings of German reunification, for instance, high-risk subsidies were one of the incentives to promote investment in the former GDR. The transfer flow $\mathrm{d}\Theta(t)$ is specified by

$$\mathrm{d}\Theta(t) = \theta_d r_K K(t)\,\mathrm{d}t + \theta_s K\,\mathrm{d}z_K. \tag{3}$$

r_K denotes the pre-transfer expected return on physical capital while $\mathrm{d}z_K$ represents the corresponding stochastic process.

The households have two options to invest, either in physical capital or in government bonds. Budget deficits are financed by issuing perpetuities paying an instantaneous riskless nominal interest rate measured in units of output. Note that $B(t)$ is not necessarily positive, as the government can become a net creditor to the public. Throughout the paper, we will assume that the government is able to precommit itself to a given policy, announced and immediately effective at $t = 0$. Thus, it will not be necessary to address time-consistency issues. Wealth $W(t)$ of a single agent is the sum of the holdings of the two assets:

$$W(t) = B(t) + K(t). \tag{4}$$

The initial value of physical capital, $K(0) = K_0$, and the number of bonds, b_0, are exogenously given whereas the initial market value of government bonds $B(0) = p(0)b_0$ is endogenously determined. Due to the productivity shock the market price of bonds as well as factor incomes are random.

The economy is populated with many identical, infinitely lived households, characterized by a time-separable utility function in consumption only. The agent maximizes her intertemporal expected utility according to the following program taking prices, tax and transfer rates as given:

$$\max_{C,n,W} E_0 \int_0^\infty U[C(t)]\mathrm{e}^{-\beta t}\,\mathrm{d}t \tag{5}$$

$$\text{s.t. } \mathrm{d}W = \{[(1-n)r_B + n(1+\theta_d)r_K]W + \omega - (1+\tau)C\}\,\mathrm{d}t + \mathrm{d}w, \tag{6}$$

with $K(0) > 0$, $y(0) = 0$. E_0 is the expectations operator conditional on the information at time $t = 0$ and β is the rate of time preference, positive by assumption. The expected returns to financial and physical capital are given by r_B and r_K, while ω denotes the expected wage rate. The corresponding stochastic processes are $\mathrm{d}z_K$, $\mathrm{d}z_B$, $\mathrm{d}z_L$, which determine the stochastic wealth process

$$\mathrm{d}w = W((1-n)\,\mathrm{d}z_B + n(1+\theta_s)\,\mathrm{d}z_K) + \mathrm{d}z_L.$$

The portfolio share of physical capital is given by n. Consumption $C(t)$ is assumed to be instantaneously deterministic. The current period utility function $U[C(t)]$ is strictly concave and of the isoelastic form:

$$U[C(t)] = \begin{cases} \dfrac{C(t)^{1-\rho}}{1-\rho} & \text{if } \rho > 0,\ \ \rho \neq 1, \\ \ln C(t) & \text{if } \rho = 1. \end{cases} \tag{7}$$

The parameter ρ denotes the Arrow/Pratt-index of relative risk aversion and is assumed to be constant.

3. Macroeconomic equilibrium

We will now proceed with the description of the competitive equilibrium allocation conditional on given policy parameters, while the analysis of optimal fiscal policy will be deferred to the next section.

Under the assumption of isoelastic preferences—in a risky environment represented by constant relative risk aversion—the optimal household behavior in the market equilibrium displays two well-known characteristics: First, consumption and wealth grow at a common rate. [2] Consequently, the propensity to consume out of wealth μ will be constant:

$$C(W,t) = \mu W(t). \tag{8}$$

Second, the optimal portfolio allocation is invariant with respect to wealth, which implies constant portfolio shares on the steady state growth path.

We assume perfect competition in the factor markets. The factor returns can then be determined by using the first-order conditions of the firm problem. Additionally, in equilibrium, the stock of knowledge equals the economy-wide stock of capital. Then, the expected wage rate and the expected rental rate of physical capital are given by the usual marginal productivity conditions, that is,

$$r_K = \alpha\gamma \quad \text{and} \quad \omega = (1-\alpha)\gamma K. \tag{9}$$

Eq. (9) displays the well-known result of Romer (1986). Due to the capital externality, the private return to capital falls short of the social return. Because the aggregate productivity shock is the only source of uncertainty in the economy, the stochastic processes of the factors of production, $\mathrm{d}z_K$ and $\mathrm{d}z_L$, are perfectly correlated with the output shock.

Market clearing requires $\mathrm{d}K = \mathrm{d}Y - C\mathrm{d}t$. From this follows immediately the stochastic accumulation equation of the capital stock. Aggregate capital evolves according to

$$\frac{\mathrm{d}K}{K} = \left(\gamma - \frac{\mu}{n}\right)\mathrm{d}t + \gamma\sigma\,\mathrm{d}y. \tag{10}$$

In order to derive the equilibrium conditions, it is necessary to establish the public sector budget constraint. The government deficit or surplus is the residual from tax receipts and transfers net of interest payments. We employ Eqs. (2), (3), (8) and (9) in order to describe the stochastic growth rate of the market value of government bonds:

$$\frac{\mathrm{d}B}{B} = \left(r_B + \frac{\theta_d\alpha\gamma n}{1-n} - \frac{\tau\mu}{1-n}\right)\mathrm{d}t + \mathrm{d}z_B + \frac{\alpha\gamma\sigma\theta_s n}{1-n}\,\mathrm{d}y. \tag{11}$$

The value of the outstanding stock of debt must equal the present discounted value of the expected flow of present and future primary surpluses. The opposite argument applies if the public sector runs a budget surplus.

[2] The details of optimization are given in the appendix.

The tax revenues from the consumption tax are instantaneously deterministic whereas subsidy payments vary with the realizations of the technology shock. In order to balance the government budget at each instant of time, the value of outstanding government bonds has to be stochastic too. Given the assumptions of a sure nominal interest rate and a fixed initial stock of bonds issued, as stated in the previous section, this implies a stochastic market price of bonds which entirely absorbs the impact of the technological disturbances. Altogether, the real rate of return on bonds is random. If, for instance, a positive realization of the productivity shock requires a large amount of subsidy payments, the market price of bonds adjusts correspondingly. Hence, it does not become necessary to change the volume of emitted bonds.

The stochastic process of the market value of government debt, $\mathrm{d}z_B$, is then determined endogenously by the constant portfolio share of bonds together with Eq. (10). The first-order conditions of the consumer optimization problem, the conjecture for optimal consumption (8) and the asset pricing relationship can now be employed to derive the equilibrium real rate of return on bonds (see (A.2) and (A.3) in the appendix):

$$r_B = \alpha\gamma(1+\theta_d) + \rho\gamma^2\sigma^2\left(1-\alpha-\frac{\alpha\theta_s}{1-n}\right) \quad \text{for } n \neq 1, \tag{12a}$$

$$r_B = \alpha\gamma(1+\theta_d) - \rho\alpha\gamma^2\sigma^2 \quad \text{for } n = 1. \tag{12b}$$

From (12) follows that the expected net-rates of return on equity and bonds differ to the amount of the risk premium which is negative, if physical capital is the riskier asset and vice versa. In this context it is important to stress that a sure nominal interest rate on bonds not necessarily implies the asset to be less risky. As already mentioned above, the stochastic market value of government bonds additionally has to be taken into account. If the government decides not to subsidize transitory capital incomes, that is $\theta_s = 0$, the limiting case $n = 1$ may occur. Eq. (12b) corresponds to the situation where the agents invest entirely in physical capital. In this case, the equilibrium value of the expected interest rate on bonds is derived via an arbitrage argument. The interest rate r_B then takes on a value at which no agent would be willing to invest in government bonds. They regard bonds as an unattractive asset and the market value of outstanding bonds is zero.

Given the functional forms of technology (1) and of instantaneous felicity (7) it is now possible to obtain closed-form solutions for the optimal consumption and portfolio choice:

$$\begin{aligned}(1+\tau)\mu = {} & \frac{\beta}{\rho} + \frac{\rho-1}{\rho}\alpha\gamma(1+\theta_d) + (1-\alpha)\gamma n \\ & + \gamma^2\sigma^2\left[(1-\rho)\alpha\theta_s + \rho(1-n)(1-\alpha) + \alpha - \frac{\rho+1}{2}\right],\end{aligned} \tag{13}$$

$$n = \frac{\beta - (1-\rho)(\psi - \frac{1}{2}\rho\gamma^2\sigma^2)}{\gamma(\tau+\alpha) - (1+\tau)\psi + \rho\gamma^2\sigma^2(1-\alpha)}. \tag{14}$$

These equations show that in equilibrium the portfolio shares and the propensity to consume out of wealth are time-invariant functions of the underlying parameters, if in addition the expected growth rate of the economy, $\psi = E(\mathrm{d}W)/(W\mathrm{d}t)$, is also constant. From the expression for the consumption–wealth ratio (13) it becomes obvious that the optimal consumption and portfolio choice are interdependent. Even in the limiting case of logarithmic preferences, that is $\rho = 1$, the optimal value of the propensity to consume is affected by the portfolio share n. This result can explicitly be attributed to the presence of labor income risk in this model. [3]

The optimal portfolio share of physical capital (14) can be expressed as a function of the expected growth rate ψ. For any change in tax-transfer policy the portfolio share adjusts correspondingly. The portfolio share of financial wealth can be obtained residually and is allowed to be of either positive or negative sign. With the optimal choice of n, the initial value of wealth and the initial market value of government bonds in terms of the initial capital stock K_0 and the endogenously determined portfolio shares can be determined:

$$W(0) = \frac{K_0}{n} \quad \text{and} \quad B(0) = \frac{1-n}{n}K_0. \tag{15}$$

In case of $n > 1$ the government is net-lender to the public. The present value of primary surpluses has to suffice to pay off existing debt, which corresponds to $B(0) < 0$. Otherwise, the intertemporal government solvency constraint will not be met. The government uses its interest income from being a creditor to the private sector to finance its continuous deficit. With financial wealth growing at the constant expected growth rate, this implies a negative stock of bonds at all points of time. Hence, the insights from debt policy as discussed by Turnovsky (1996) for the deterministic setting or Corsetti (1997) apply. The case of Ponzi-games can be ruled out, if the standard transversality condition for dynamic optimization problems is met. [4]

From the market clearing condition (6), the expected steady state growth path can be obtained as follows:

$$\psi = \frac{1}{\rho}[\alpha\gamma(1+\theta_d) - \beta] + \gamma^2\sigma^2\left[\frac{\rho+1}{2} - \alpha(1+\theta_s)\right]. \tag{16}$$

The expected growth rate of the economy is the sum of two components: The first resembles the growth rate of the deterministic Arrow–Romer model while the second reflects the agent's optimal response to technological risk. [5] The expected growth rate will exceed the deterministic growth rate if the agent has a motive for precautionary saving, i.e. if she is sufficiently risk averse. From this can be seen that current shocks may have long-lasting effects on the macroeconomic trend. Apart from the

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[3] In the presence of a pure capital risk, the propensity to consume would reduce to $(1+\tau)\mu = \beta$ for logarithmic preferences. The intertemporal income and substitution effects exactly offset in this case, and the propensity to consume is unaffected by risk. This case is often referred to as 'certainty equivalence'.

[4] See eq. (A.5) in the appendix.

[5] Obviously, the growth rate of the deterministic setting corresponds to the case $\sigma = 0$.

individual risk and time preferences the subsidy on permanent capital income θ_d as well as the subsidy on transitory capital returns θ_s affect expected growth. The response of the expected growth rate to a change in the policy parameters is given by

$$\frac{\partial\psi}{\partial\theta_d} = \frac{1}{\rho}\alpha\gamma > 0, \tag{17a}$$

$$\frac{\partial\psi}{\partial\theta_s} = -\alpha\gamma^2\sigma^2 < 0, \tag{17b}$$

$$\frac{\partial\psi}{\partial\tau} = 0. \tag{17c}$$

Net-capital returns determine the amount of individual savings and hence the long-run growth rate of the economy. By subsidizing capital returns, the mean as well as the volatility of capital income are affected. The positive growth effect of a rise in θ_d corresponds to the economic channel that eliminates the external effect in the deterministic setting. The real rate of return on physical capital increases, causing investment to be more attractive. In contrast to this, a rise in θ_s raises the variance and the associated risk of the return on capital. Therefore, the fraction of physical capital in the portfolio of assets declines, which implies lower expected growth. In terms of Rothschild and Stiglitz (1970), this case represents a mean preserving spread which causes physical capital to be less attractive.

The results of Domar and Musgrave (1944) and Stiglitz (1969) regarding the taxation of risky assets apply. They pointed out that taxation with full loss-offset provision can actually increase the demand in risky assets. In the context of investment subsidies (i.e. negative tax rates) considered here, clearly the opposite holds.

The partial derivative (17c) shows that the consumption tax does not affect accumulation. This is a well-known result from the analysis of tax incidence in dynamic representative agent models. Here, labor supply is inelastic and the consumption tax affects the consumption opportunities of all periods to the same extent. For these reasons, there neither is a distortion of the relative price between consumption and saving nor a distortion of the labor–leisure choice. The consumption tax amounts to a lump-sum tax.

Other things equal, an increase in the consumption tax rate reduces the value of the propensity to consume out of wealth (13), thereby reflecting a decrease in the consumption opportunities. This naturally would have a negative impact on lifetime utility in a partial equilibrium context. Nevertheless, in a dynamic general equilibrium model both sides from the government budget constraint have to be taken into account. Consequently, a rise in tax revenues is always accompanied by an increase in government expenditures. This increases long-run consumption opportunities either by (17a) or by a corresponding adjustment in the bond return (12).

In order to discuss the macroeconomic effects of alternative policy designs from a welfare point of view, it is necessary to consider lifetime utility of a representative agent as specified by (5) evaluated along the competitively chosen path. By (8), indirect utility depends on individual wealth. In equilibrium, wealth is log-normally distributed and follows a random walk with positive drift. Starting from initial wealth $W(0)$ at time 0, time t wealth is given by

$$W(t) = W(0)\mathrm{e}^{(\psi - \frac{1}{2}\gamma^2\sigma^2)t + \gamma\sigma y(t)}. \tag{18}$$

This finally enables us to derive lifetime utility. According to the closed-form solutions describing the macroeconomic equilibrium, (13), (14) and (16) as well as the initial values (15), welfare depends on the propensity to consume out of wealth, on the expected growth rate, and on the optimal portfolio. Hence, the effect of any policy on welfare can be assessed in terms of its impact on (i) the propensity to consume out of wealth, (ii) the portfolio share of physical capital, and (iii) the expected growth rate of the economy. Ceteris paribus, individual welfare increases with a rise in expected growth or in the consumption–wealth ratio. An increase of the portfolio share of capital is equivalent to a decrease in the demand for government bonds. This induces a devaluation of initial wealth as can be seen from (15) and finally leads to a reduction of lifetime utility.

We utilize the market clearing condition (10) and substitute the consumption–wealth ratio for $\mu = n(\gamma - \psi)$. Individual welfare is then given as follows:

$$V[W(0), 0] = \frac{K_0^{1-\rho}(\gamma - \psi)^{1-\rho}}{(1-\rho)[\beta - (1-\rho)(\psi - \frac{1}{2}\rho\gamma^2\sigma^2)]} \tag{19}$$

and $V[W(0), 0] = [\beta \ln(\gamma - \psi) + \psi - \frac{1}{2}\gamma^2\sigma^2 + \beta \ln K_0]/\beta^2$ for logarithmic preferences respectively.

Since labor is assumed to be inelastically supplied, leisure is not an argument in the utility function and there are no transitory dynamics. As already mentioned above, any tax or transfer policy is immediately effective at time $t = 0$. For this reason it is possible to draw direct comparisons between alternative steady states and the associated welfare.

4. Optimal policy

In the preceding section we demonstrated that the growth effects of a change in the transfer rates on the permanent and transitory capital returns are counteracting. Hence, extending the learning-by-doing framework with technological uncertainty gives rise to several questions concerning the design of an optimal tax-transfer scheme: First, should permanent and transitory capital incomes be treated uniformly or differently? Are risk-associated subsidies generally optimal? What are the consequences for a welfare-maximizing fiscal policy, if transitory incomes are not observable? Is the structure of government revenues crucial for the design of an optimal policy?

From the literature on modern growth theory it is well-known that within the Romer (1986) setting individual optimization in the decentralized economy goes along with a Pareto-inferior allocation. This result in general continues to hold in the stochastic model, but with one additional feature. As the stock of knowledge is regarded as a constant within individual optimization, the expected marginal product of capital falls short of the social return. This outcome is captured by the first term of the expected growth rate (16). But moreover, the second term reflects that the agents also underestimate the riskiness of capital incomes. For this reason, risk-induced accumulation is too high. A correct perception of the volatility of capital returns, would imply a larger intertemporal substitution effect, thereby causing lower expected growth. The two components of the expected growth rate sum up to a situation where either growth of the competitive economy is suboptimally low or inefficiently large compared to the Pareto-efficient allocation, the final outcome depending on whether the first or the second effect dominates. This property of the stochastic model is independent of any specific policy the government undertakes.

In general, in the presence of human capital externalities, the socially optimal path can only be achieved by means of fiscal intervention. In a deterministic world the policy parameters chosen in this paper, i.e. an investment subsidy financed by revenues from a consumption tax, are a sufficient policy mix to achieve Pareto-optimality. But, as we demonstrated above, the external effect as well as the subsidy has an ambiguous impact on expected growth and consequently on welfare.

In a first step we will now characterize the conditions necessary to attain a first-best allocation within the stochastic setting. The welfare effects of a change in the differential transfer rates θ_i, $i = d, s$ can be derived as follows:

$$\frac{\partial V(0)}{\partial \theta_i} = \frac{K_0^{1-\rho}\rho(\gamma - \psi)^{-\rho}}{[\beta - (1-\rho)(\psi - \frac{1}{2}\rho\gamma^2\sigma^2)]^2} \frac{\partial \psi}{\partial \theta_i}(\psi^* - \psi), \tag{20}$$

where ψ^* denotes the Pareto-optimal expected growth rate. [6] From (20) it is possible to derive a condition for the design of optimal policies. The first term is positive for feasible solutions of the model, because we require the propensity to consume and likewise the portfolio share of capital to be positive. The second term depicts the growth effects already discussed in Eqs. (17a) and (17b). It is of positive sign in case of an increase in θ_d and negative with a rise in θ_s. The third term reflects the gap between competitive and Pareto-optimal growth and includes the case that risk as well as fiscal policy may lead to either excessively high or too low growth.

Individual welfare increases (decreases) with a rise in the subsidy on permanent (transitory) capital income, if the competitively chosen growth rate falls short of the Pareto-optimal one and vice versa:

$$\left.\begin{array}{l} \frac{\partial V(0)}{\partial \theta_d} \lesseqgtr 0 \\ \frac{\mathrm{d}V(0)}{\partial \theta_s} \gtreqless 0 \end{array}\right\} \Longleftrightarrow \psi \gtreqless \psi^*. \tag{21}$$

[6] The socially optimal growth rate is derived by eliminating the external effect. In the set-up considered here this is equivalent to the case of $\alpha = 1$ in (16).

From this follows immediately that the impact of any public policy on welfare is entirely determined by its growth effects. The specific structure of government revenues is of minor importance which reflects the typical Ricardian equivalence result in a general equilibrium context. In order to maximize individual welfare, fiscal policy has to be chosen in a way that the competitive growth rate equals the optimal one. Under this condition, the general rule for an optimal policy can be expressed as

$$\psi \overset{!}{=} \psi^* \Rightarrow \theta_d^* = \rho\gamma\sigma^2\left(\theta_s^* - \frac{1-\alpha}{\alpha}\right) + \frac{1-\alpha}{\alpha}. \tag{22}$$

Eq. (22) displays one main result of our paper: The two transfer rates cannot be chosen independently. An optimal policy scheme is characterized by a policy rule that expresses one of the subsidy rates as a function of the other, $\theta_d^* = f(\theta_s^*)$, with $d\theta_d^*/d\theta_s^* > 0$. A higher subsidy on random income parts is to be accompanied by an equivalent increase in the subsidy on permanent capital returns. This result can be explained with regard to the growth effects: Starting from a situation $\psi < \psi^*$, a rise in θ_s increases the riskiness of capital returns. This causes to a decline in the expected growth rate, thus moving it away from the Pareto-optimal one. This negative effect has to be outweighed by an increase in θ_d which stimulates accumulation due to a rise in expected capital income. The reverse argument applies for the case of $\psi > \psi^*$.

The opposite growth effects indicate that the two transfer rates may be set uniformly, but that this is not necessarily required for an optimal policy scheme. As can be seen from Eq. (22), the interdependence between the subsidy rates leads to a continuum of feasible optimal policies which is displayed in Fig. 1. The solid line represents the optimal linear combination θ_d^*, θ_s^* in case of $\rho\gamma\sigma^2 < 1$, that is relatively

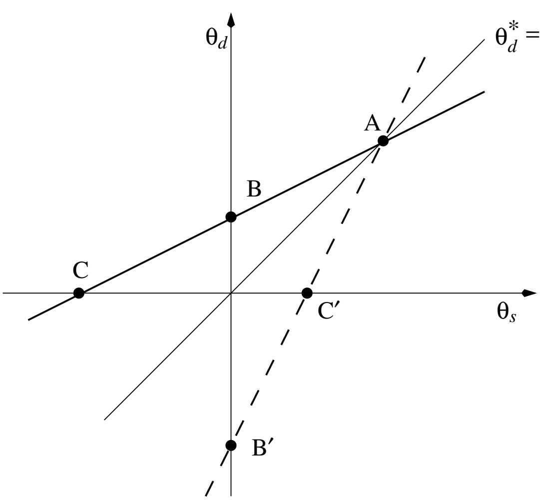

Fig. 1. Optimal transfer-schemes.

low risk. The dashed line displays the corresponding optimal subsidy rates for $\rho\gamma\sigma^2 > 1$.

If we now refer to the first question of this section, it is possible to show that the economy is capable of attaining a welfare maximum with uniform transfer rates, but furthermore, that there exists a continuum of optimal policies, all of them characterized by differing subsidy rates.

In the following, we will discuss three policies within this continuum. Policy A reflects the case of a flat-rate subsidy and can be viewed as a straightforward extension of the deterministic model. Points B, B′ and C, C′ of Fig. 1 correspond to optimal policies where either one of the transfer rates is set to zero. It is the special feature of the stochastic model that government may achieve Pareto-optimal growth by subsidizing investment in accordance with the associated risk. If, for instance, the government accounts especially for high-risk investment, stochastic capital returns have to be subsidized at a higher rate than the deterministic ones. Policy B, B′ focuses solely on a subsidy on permanent (risk-less) capital incomes, whereas policy C, C′ is the polar case of a subsidy exclusively on transitory (risky) capital returns.

4.1. Flat-rate subsidy

We will now discuss the question of optimal policy if the government cannot discriminate between transitory and permanent income parts and consider this as a benchmark case. We solve for a flat-rate subsidy as in Corsetti (1997), that is $\theta_d = \theta_s = \theta$.

The optimal subsidy can be determined as

$$\theta^* = \frac{1-\alpha}{\alpha}. \tag{23}$$

This outcome confirms the well-known result from the deterministic model. A production subsidy at the rate θ^* completely offsets the distortions induced by the knowledge spillover. It is unambiguously positive. The externality from human capital affects accumulation twofold: It is present in the expected capital incomes as well as in capital volatility. On the one hand, the agents underestimate average capital returns. This by itself leads to suboptimally low growth and is compensated by a positive subsidy on deterministic income parts. On the other hand, due to the externality, the agents equally underestimate the riskiness of capital returns and ceteris paribus accumulate too much. The subsidy on random income components offsets this effect of overaccumulation and depresses growth. Despite the counteracting growth effects of the two subsidies when considered separately, in the case of the optimal flat-rate policy, these effects finally sum up to optimal growth.

4.2. Subsidy/tax on permanent capital returns

The next issue we address is, whether there exists a policy capable of supporting the first-best allocation, if we exclude subsidies on transitory capital returns from the menu of admissible policy instruments. This situation might arise if the government cannot perfectly monitor stochastic income parts.

Moreover, in an economy where either budget deficits or surpluses are not politically feasible or changes in labor income taxes cannot be forced through, the optimal policy presented here is the only one that completely internalizes the distortion in human capital accumulation. This is due to the fact that consumption as well as tax revenues are instantaneously deterministic. A government budget balanced at each instant of time with a market value of public debt equal to zero would then require deterministic expenditures. Consequently, transitory capital returns have to be ruled out from subsidization.

From the assumption $\theta_s = 0$ and Eq. (22) the optimal subsidy on permanent capital incomes can be derived as follows:

$$\theta_d^* = \frac{1-\alpha}{\alpha}(1-\rho\gamma\sigma^2). \tag{24}$$

The optimal subsidy rate can be positive or negative. In the first case, if the impact of the technology shock is not too large, the premium on risky capital returns is small enough to preserve the general results of the deterministic setting. The growth diminishing effect of the externality offsets the growth enhancing effect. Accumulation in total is suboptimally low and welfare maximizing growth is achieved by means of subsidizing investment in physical capital.

The second case reflects a high-risk situation. As the agents underestimate the riskiness of capital returns due to the knowledge externality, investment of the competitive economy exceeds the Pareto-optimal level. Hence, the expected growth rate is suboptimally high. In order to maximize welfare it is necessary to impose a tax on physical capital, that is $\theta_d^* < 0$.

An important result is that the optimal subsidy is smaller than the uniform rate derived in the preceding section as can be seen from point B in Fig. 1. This result can be explained if we take the growth-depressing effect of θ_s into account. This effect is inactive in the context considered here, hence subsidization can take place at a smaller rate in order to accomplish optimal growth and welfare. Additionally, total expected subsidy payments are lower within this policy design compared to expected payments with uniform transfer rates. Expected subsidy payments even fall short of payments necessary to completely internalize the knowledge spillover in the absence of technological uncertainty, thereby implying a lower government expenditure share. If instead permanent capital returns were to be subsidized at the higher uniform rate $\theta^* = (1-\alpha)/\alpha$, the distortions would not be completely offset. In this context the equilibrium growth rate would remain suboptimally high accompanied by the corresponding Pareto-inferior welfare level.

4.3. Subsidy/tax on transitory capital returns

The result from the preceding section suggests itself to discuss the opposite case given by points C, C′ in Fig. 1 where permanent capital returns are excluded from subsidization, that is $\theta_d = 0$. Now, the government focuses solely on transitory capital incomes. This policy could contain for example subsidies on risky projects at higher rates, in order to promote R&D and human capital accumulation.

The optimal transfer rate on stochastic capital income is then given by

$$\theta_s^* = \frac{1-\alpha}{\alpha}\left(1 - \frac{1}{\rho\gamma\sigma^2}\right). \tag{25}$$

Taking the argument from above, the sign of the optimal subsidy rate can be negative or positive as well. If the volatility is sufficiently low, the term in brackets in (25) is negative. In this specific case the optimal policy is to tax stochastic income components. This result reflects the argument well-known from the literature on taxation under uncertainty, as given by Domar and Musgrave (1944), Stiglitz (1969) or more recently Smith (1996b), Turnovsky (1999a), Ott and Soretz (in press) and Soretz (2004). Taxation of risky returns lowers the volatility of future income streams. A risk averse agent responds to a less risky environment by increasing accumulation, accompanied by a rise in the expected growth rate.

Contrary, consider the case where the impact from the technological disturbance is strong enough. Now, the growth enhancing effect from the underestimation of the riskiness of capital return dominates. The government then has to subsidize transitory capital incomes in order to rise the volatility of capital incomes, which ultimately induces a decrease in accumulation. In short, subsidization of transitory incomes drives the suboptimally high expected growth rate towards the Pareto-optimal level.

4.4. Changes in revenue policies

The analysis of government expenditure undertaken so far has shown that there is a continuum of feasible policies capable of attaining the Pareto-optimal state. We discussed three benchmark policies. Let us now turn towards the last question which was concerned with the specific role of government revenues. The optimality condition (21) states that any public policy can be assessed via its growth effects. Neither the consumption tax nor the government debt are arguments of the growth rate as we excluded the labor–leisure choice from our analysis. Hence, the expenditure side of government activity is separable from the revenue side and Ricardian equivalence holds. Each optimal subsidy-scheme can be financed via an arbitrary mix of consumption taxes and government debt.

The optimal policy schemes also include the two limiting cases $\tau = 0$ and $n = 1$. From the government budget constraint (11) follows, that subsidy payments are financed by revenues from taxing consumption as well as by issuing bonds. A fiscal policy without government debt, that is $n = 1$, was already discussed above. In this case, the stochastic income components cannot be subsidized and the optimal expenditure policy is described by Eq. (24). Now, the expected real rate of return on government bonds represents the certainty equivalent of capital return and is determined by the arbitrage condition (12b).

In the following, we focus on the second limiting case of $\tau = 0$. Ruling out a consumption tax does not necessarily imply that it is not possible to design an optimal policy which completely offsets the external effect of accumulation. The only difference is that the subsidies now are completely financed by government bonds. From (15), the equilibrium value of bonds is implicitly given by the portfolio share of physical capital, which finally can be determined according to (14), (16) and (22).

$$n = \frac{\beta - (1-\rho)(\psi - \frac{1}{2}\rho\gamma^2\sigma^2)}{\beta - (1-\rho)(\psi - \frac{1}{2}\rho\gamma^2\sigma^2) - (1-\alpha)\gamma(1-\rho\gamma\sigma^2)}. \tag{26}$$

In case of a low-risk situation, the portfolio share exceeds unity. This immediately becomes obvious, if we consider the last term in the denominator which decides upon the size of n. For sufficiently low risk this term is positive and the government is net-creditor to the public. The subsidy payments are totally financed from interest revenues. In so far, our findings confirm results Turnovsky (1996) derived for the deterministic setting and Corsetti (1997) discussed in the presence of income taxes.

Contrary, if volatility is sufficiently high, the respective term of the denominator is negative and the portfolio share lies in the unit interval. Without public policy, risk-induced accumulation is too high and the competitively chosen growth rate exceeds the optimal one. Consequently, in an optimal policy, the government has to fix the subsidies such as to downsize the growth rate. This, in combination with a zero consumption tax, leads to a positive value of government debt.

5. Conclusion

In this paper we developed a stochastic endogenous growth model with spillover effects of technical knowledge. Assuming differential subsidies on permanent and transitory capital returns, we examined alternative designs for tax-transfer schemes. The analysis was motivated by the idea that feasible sets of policy instruments well-known from the deterministic framework cannot be carried over to the stochastic setting without further refinement.

The knowledge externality affects growth twofold under productivity risk. On the one hand, the agents underestimate mean capital returns and invest suboptimally low. On the other hand, the agents likewise underestimate capital volatility and invest suboptimally high. Altogether, growth of the competitive economy might be either too high or too low, depending on the relative magnitude of risk.

Our attention was especially focused on different designs for optimal policy schemes. We found that subsidizing production does not only affect the mean but also the volatility of future capital income flows. We demonstrated that a subsidy on permanent capital returns displays the well-known effects from the deterministic setting and enhances growth. But contrary, a risk averse individual would respond to transfers on transitory capital returns with a decrease in savings. Consequently, subsidizing random incomes depresses growth. These ambiguous growth effects carry over to the analysis of welfare and imply a continuum of optimal policies. We demonstrated that granting a subsidy is not generally optimal, the results depending on the impact of risk. It may be optimal for the government to depress growth by means of taxing deterministic income.

We focused on several benchmark cases and found that total expected subsidy payments for a complete offset of the distortion are lower, if only permanent capital incomes are permitted to be subsidized. In this case the subsidy payments even fall short of the spending necessary in the deterministic framework. In the opposite case, the exclusive treatment of transitory capital incomes, the optimal policies reverse in sign. If growth originally is too high due to a high-risk situation, the government in fact has to subsidize transitory capital incomes in order to reduce accumulation.

Finally, we demonstrated that from the viewpoint of optimal policy the way of financing government spending, either via consumption tax revenues or via issuing bonds is of minor importance. In the general equilibrium context considered here all instruments of public policy enter into individual optimization. Hence, even if we exclude the possibility of taxing consumption an optimal policy can be found.

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Appendix

The value function $V[W(t),t]$ denotes the maximum feasible level of lifetime utility. Positing the time-separable form $V[W(t),t] = e^{-\beta t}J[W(t)]$, the stochastic differential of the value function can be derived by application of Itô's Lemma:

$$\mathscr{L} = e^{-\beta t}\Big\{U(C) - \beta J + J'(W)\{[(1-n)r_B + n(1+\theta_d)r_K]W + \omega - (1+\tau)C\} + \frac{1}{2}J''(W)\sigma_W^2\Big\} \tag{A.1}$$

with the variance of wealth given by $\sigma_W^2 = E(\mathrm{d}w)^2/\mathrm{d}t$.

The optimality conditions of the above problem with regard to C, n and W are

$$0 = U'(C) - (1+\tau)J'(W), \tag{A.2}$$

$$0 = J'(W)[(1+\theta_d)r_k - r_B]W + \frac{1}{2}J''(W)\frac{\partial\sigma_W^2}{\partial n}, \tag{A.3}$$

$$0 = J'(W)[(1-n)r_B + n(1+\theta_d)r_K - \beta] + \frac{1}{2}J''(W)\frac{\partial\sigma_W^2}{\partial W} + J''(W)\{[(1-n)r_B + n(1+\theta_d)r_K]W + \omega - (1+\tau)C\} + \frac{1}{2}J'''(W)\sigma_W^2. \tag{A.4}$$

Eq. (A.2) reflects the well-known result from intertemporal optimization, that is marginal utility weighted with the tax factor is equalized over time. It determines the accumulation process together with (A.4). The expression (A.3) is the standard first-order condition of a portfolio problem in an intertemporal C-CAPM when all returns are perfectly correlated. The optimal time-paths for consumption and the portfolio shares are functions of the derivatives of the value function and form a stochastic differential equation in $J(W)$. Hence, a function $J(W)$ has to be found that solves the first-order conditions and maximizes the stochastic integral (5).

Furthermore, feasibility of an intertemporal consumption program requires the following transversality condition to be satisfied. Otherwise, as demonstrated by Merton (1969), expected utility would not be bounded

$$\lim_{t\to\infty} E_t\left[J(W)\mathrm{e}^{-\beta t}\right] = 0. \tag{A.5}$$

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